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EXTENDED APPLICATIONS OF THE HOT-WIRE ANEMOMETER

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SUMMARY

Two new fields of application of the hot-wire anemometer are proposed, and the appropriate response equations and measuring procedures are developed.

The first analysis leads to a method for the measurement of physically significant statistical quantities in a turbulent flow with heat transfer; for example, the turbulence levels, the temperature fluctuation level, the turbulent heat-transfer coefficient, the velocity scale, the temperature scale, and some spectrums.

The second analysis involves the use of the hot-wire in the turbulent isothermal mixing of two appropriately different gases. If the thermal conductivity of the mixture is known and is a monotonic function of the relative concentration, it is possible to measure the mean velocity and mean concentration at any point. If no data are available on the thermal conductivity of the mixture, this additional unknown can be determined by an additional measurement. Furthermore, it is also possible to measure the various statistical functions of the fluctuating velocities and the local concentration fluctuation, provided, again, that the thermal conductivity is a known monotonic function of the concentration.

Although the details of the present analysis are dependent upon the accuracy of King's equation for the rate of heat loss from fine wires, the general approach is equally valid for any possibly more accurate equation that may be deduced.

INTRODUCTION

The hot-wire anemometer has been used for many years in the measurement of the mean velocities of flowing gases (reference 1) and, more recently, in the measurement of various statistical quantities associated with the random velocity fluctuations occurring in turbulent flow (references 2 to 7 and others). It has also been used as a microphone (reference 8). This anemometer consists essentially of an

electrically heated wire of extremely small mass, the temperature of which varies in response to rapid changes in the instantaneous flow (cooling) velocity. If the heating current is maintained constant, the resulting voltage fluctuations, the intensity of which is expressible as a function of the velocity fluctuations, may be amplified and measured. Attenuation and phase lag, due to the nonzero heat capacity of the wire and the finite rate of electrical heating, may be compensated by a suitable network between two of the amplifier stages. The essentials of the technique are described in reference 2.

In a detailed consideration of turbulent fluid flow, there are several statistical functions of the randomly fluctuating velocities that are of importance. The simplest of these are the following:

(a) The three components of the turbulence level or intensity $\frac{u'}{U}$, $\frac{v'}{U}$, and $\frac{w'}{U}$ which, squared and added, give the ratio of kinetic energy of random fluctuation to directed kinetic energy.

(b) The double correlation functions, that is, the statistical correlation between two components at separate points; for

example, $R_2 = \frac{\overline{u_1 u_2}}{\overline{u_1'} \overline{u_2'}}$. Von Kármán (reference 9) and Von Kármán and

Howarth (reference 10) have shown that only two of these functions are independent for isotropic turbulence.

(c) The energy spectrum of the turbulent fluctuations. Taylor (reference 11) has demonstrated that this is the Fourier transform of one of the two principal components of the correlation tensor.¹

(d) The scale of turbulence, defined by Taylor (reference 12) for isotropic turbulence as the integral of one of the double correlation

functions; for example, $L = \int_0^\infty R_2(y) dy$.

(e) The microscale of turbulence, identified with the rate of dissipation of turbulent energy into heat, the size of the smallest eddies, was also defined by Taylor (reference 12), who showed this to be the abscissa intercept of the vertex-tangent parabola of the $R_2(y)$ curve in isotropic turbulence.

¹In low-intensity isotropic turbulence.

(f) The turbulent shear stress $\tau = -\rho \overline{uv}$ was first identified by Osborne Reynolds in his classic formulation of the equations of motion for turbulent flow (reference 13). This stress is, of course, a measure of the lateral rate of momentum transfer or diffusion.

All the foregoing functions have been measured for various turbulent flows (references 6, 7, 14, 15, 16, 17, 18, 19, and others), principally with the hot-wire technique, although the most detailed measurements have of necessity been confined to investigations of the "isotropic" turbulence far behind grids placed in a uniform air stream.

There are other kinematic quantities of considerable interest:

(g) Batchelor and Townsend (reference 20) have recently measured three statistical functions of the time derivatives of the velocity

fluctuation $\overline{\left(\frac{\partial u}{\partial t}\right)^2}$, $\overline{\left(\frac{\partial^2 u}{\partial t^2}\right)^2}$, and $\overline{\left(\frac{\partial u}{\partial t}\right)^3}$. These enter into terms describing the time rate of change of vorticity in isotropic turbulence.

(h) The rate of lateral kinematic diffusion of turbulent energy in a turbulent shear flow $(\sim \rho u^2 v)$ has apparently not been measured yet, although suitable techniques are fairly well known.

(i) The pressure fluctuation diffusion term $(\sim \overline{pv})$ in the turbulent energy equation has not been measured, principally because of the difficulty of obtaining an instrument that responds to static-pressure fluctuations independently of velocity fluctuations.

In a detailed consideration of turbulent motion, it is of interest to obtain information on the heat and material diffusion coefficients as well as the momentum and energy diffusion already mentioned. In fact, this information should be useful both as an end in itself and as additional evidence to be applied in obtaining a better understanding of the nature of turbulence. Consequently, it seemed worthwhile to investigate the possibilities of exploiting the hot-wire beyond the simple anemometric applications.

The rate of cooling of a body immersed in a fluid stream clearly depends upon: (a) The stream velocity, (b) the temperature difference, (c) the physical constants of the fluid, and (d) the physical constants of the wire. For the simple hot-wire anemometer, conditions (b), (c), and (d) are effectively fixed, and the instrument response measures condition (a). Changes in stream temperature certainly involve changes in condition (b) and possibly in condition (c). Thus the instrument responds to temperature changes or fluctuations. Changes in the

relative concentration of a flowing mixture of two fluids with different physical constants lead to variations in condition (c). In principle, the hot-wire instrument responds to concentration changes or fluctuations.

Considerations of algebraic and experimental complication have restricted the present treatment to cases in which fluctuations in velocity and only one of the aforementioned two quantities coexist, although this limitation is not dictated by principle.

With this experimental technique, the new quantities which it may be of interest to measure are the temperature fluctuation level $\left(\frac{\vartheta'}{\bar{\vartheta}}\right)$, the spectrum of temperature fluctuation, the heat diffusion $(\sim \vartheta \bar{v})$, the correlation coefficient $\left(\frac{\vartheta_1 \vartheta_2}{\bar{\vartheta}^2}\right)$, and the thermal scale and microscale, analogous to Taylor's kinematic scales, as well as the corresponding statistical functions of the concentration fluctuation in the turbulent mixing of two gases.

The fact that direct measurement of $\bar{\vartheta v}$ should be possible with a hot-wire instrument has been known for many years (see, for example, remarks in reference 21), but no actual work appears to have been carried out.

Distributions of momentum, heat, and material transport coefficients in a shear flow are directly calculable from measurements of the mean velocity, temperature, and concentration profiles, respectively. However, it has been found that the mean profile is relatively insensitive to phenomenological assumptions on the transfer coefficient (references 16 and 19), so that direct experimental determination of the transfer is desirable.

SYMBOLS

U	instantaneous total velocity
$\bar{U}$	mean velocity
u	instantaneous velocity fluctuation in direction of mean velocity $(U - \bar{U})$

v, w	normal components of velocity fluctuation
u', v', w'	root-mean-square values of u, v , and w
T_a	instantaneous absolute temperature of gas
$\bar{T}_a$	mean absolute temperature
θ	instantaneous temperature fluctuation $(T_a - \bar{T}_a)$
θ'	root-mean-square value of θ
T_r	absolute temperature of reference medium (e.g., the air at rest, for a free jet)
$\theta = T_a - T_r$	
$\bar{\theta} = \bar{T}_a - T_r$	
Γ	instantaneous concentration of secondary gas; in present applications, $\Gamma = \frac{\text{Mols per unit volume of the secondary gas}}{\text{Total mols, of air and gas, per unit volume}}$
$\bar{\Gamma}$	mean Γ
γ	instantaneous concentration fluctuation $(\Gamma - \bar{\Gamma})$
γ'	root-mean-square value of γ
R	instantaneous resistance of electrically heated hot-wire (unless otherwise noted)
$\bar{R}$	mean R
$r = R - \bar{R}$	
R_a	instantaneous resistance of unheated hot-wire
$\bar{R}_a$	mean R_a (resistance at $\bar{T}_a$)

$$r_a = R_a - \bar{R}_a$$

R_e instantaneous "equilibrium" hot-wire resistance;
that is, resistance hot-wire would have at any
moment if it had zero lag

$\bar{R}_e$ mean R_e

R_r resistance of wire at T_r

R_0 resistance of wire at 0°C

i heating current

e voltage fluctuation across hot-wire

l hot-wire length

d hot-wire diameter

σ_w specific resistivity of wire material

α thermal coefficient of change of resistance of
wire material

ϵ sensitivity of directional meter

ξ, ζ coefficients in assumed parabola relating thermal
conductivity to concentration in a gas
mixture

μ sensitivity of a simple hot-wire

Ω ohms (electrical resistance)

ρ_w density of wire material

m mass of wire

s specific heat of wire material

k thermal conductivity of fluid at T_a

$\bar{k}$	thermal conductivity of fluid at $\bar{T}_a$
k_a	thermal conductivity of air
k_g	thermal conductivity of secondary gas
$\Delta k = k_g - k_a$	
c_p	constant-pressure specific heat of fluid at T_a
$\bar{c}_p$	constant-pressure specific heat of fluid at $\bar{T}_a$
c_{pa}	constant-pressure specific heat of air
c_{pg}	constant-pressure specific heat of secondary gas
$\Delta c_p = c_{pg} - c_{pa}$	
ρ	density of fluid at T_a
$\bar{\rho}$	density of fluid at $\bar{T}_a$
ρ_a	density of air
ρ_g	density of secondary gas
k', c_p', ρ'	fluctuations
$\Delta\rho = \rho_g - \rho_a$	
H	heat energy in hot-wire
T	wire temperature
t	time

CONVENTIONAL HOT-WIRE-ANEMOMETER RESPONSE EQUATIONS

Following the approach of Dryden and Kuethe (reference 2), King's semiempirical form is assumed for the rate of heat loss from a wire immersed in a flowing fluid. For the time rate of increase of heat energy in the wire this gives

$$\frac{dH}{dt} = i^2 R - (A' + B' \sqrt{U})(T - T_a) \quad (1)$$

where the physical make-up of A' and B' as deduced by King is

$$A' = alk$$

$$B' = bl \sqrt{dc_p \rho} k$$

Here a and b are regarded as numerical constants, although they are dimensional, and should show some variation if a wide enough temperature range is considered.

The term $A'(T - T_a)$ presumably represents heat loss due to free convection and radiation. Therefore, it is clear that the constant a must include such physical quantities as the acceleration of gravity, the thermal expansion coefficient of the fluid, the specific heat of the fluid, a temperature function, a radiation constant, and so forth. The term $B' \sqrt{U}(T - T_a)$ represents forced convection, and, after k has been factored out, the term can be written as proportional to the product of a Reynolds number and a Prandtl number.

Equation (1) can be written as

$$\frac{4.2ms}{R_0 \alpha} \frac{dR}{dt} = i^2 R - (A + B \sqrt{U})(R - R_a) \quad (1a)$$

and if equilibrium conditions are considered such that $\frac{dR}{dt} = 0$, writing the equilibrium value of R as R_e , the following equation is obtained:

$$\frac{i^2 R_e}{R_e - R_a} = A + B \sqrt{U} \quad (2)$$

where

$$A = \frac{al}{R_0 \alpha} k$$

$$B = \frac{bl}{R_0 \alpha} \sqrt{dc_p \rho k}$$

In considerations of averaged or steady-state operation, R_e in equation (2) may be replaced by $\bar{R}$, and this equation is the usual mean velocity calibration of the hot-wire.

King deduced his equation for steady-state operation, and a primary assumption of the hot-wire-anemometer theory is that this rate of heat loss is independent of acceleration.

A convenient alternative form of equation (2) is

$$\frac{i^2 R_a}{R - R_a} = A - i^2 + B \sqrt{U} \quad (2a)$$

For the measurement of turbulence, interest is centered in the voltage fluctuation set up across the hot-wire due to a small fluctuation in velocity. The analysis for large fluctuations has proved intractable, but, as illustrated in appendix A of reference 22, the error committed in applying the small perturbation results in the measurement of rather large fluctuation levels is not necessarily excessive.

Letting

$$U = \bar{U} + u$$

$$R = \bar{R} + r$$

$$e = ir$$

where

$$\frac{u}{\bar{U}} \ll 1 \quad \text{and} \quad \frac{r}{\bar{R} - R_a} \ll 1$$

and substituting these into equation (2) yields

$$\frac{i^2(\bar{R} + r)}{(\bar{R} - R_a)\left(1 + \frac{r}{\bar{R} - R_a}\right)} = A + B\sqrt{U}\left(1 + \frac{u}{U}\right)^{1/2}$$

Keeping only the linear terms of the appropriate binomial expansions, otherwise neglecting the squares of small quantities, and substituting from equation (2) for $(A + B\sqrt{U})$, there is finally obtained

$$e = -\frac{(\bar{R} - R_a)}{2iR_a} \left[i^2\bar{R} - A(\bar{R} - R_a) \right] \frac{u}{U} \quad (3)$$

Thus, the coefficient of $\frac{u}{U}$ is the sensitivity of a single hot-wire to velocity fluctuations along the main stream direction.

Since the wire has a nonzero heat capacity and only a finite rate of electrical heating, there is a time lag in the response which normally leads to an appreciable drop in the curve of hot-wire response against frequency above some particular frequency. The characteristic "time constant" M can be computed approximately by substituting from equation (2) into equation (1a) to obtain

$$\frac{4.2ms}{R_0\alpha} \frac{dR}{dt} = \frac{i^2 R_a (R_e - R)}{R_e - R_a} \quad (4)$$

Dryden and Kuethe (reference 2) transform this further but find no useful general solution. Therefore, they use the assumption of small perturbations to justify the replacement of $(R_e - R_a)$ by $(\bar{R} - R_a)$ in the denominator of the right side, which leads to the simple form

$$\frac{dR}{dt} + \frac{1}{M}R = \frac{1}{M}R_e \quad (5)$$

or

$$\frac{d}{dt}(R - \bar{R}) + \frac{1}{M}(R - R_e) = 0$$

where the time constant

$$M = \frac{4.2ms(\bar{R} - R_a)}{i^2 R_a R_0 \alpha}$$

is the time required for $(R - \bar{R})$ to become equal to $\frac{1}{e}$ times an original step-function difference, say, $(R_1 - \bar{R})$. The definition of M is, of course, not dependent upon the assumption of small perturbation. Further details are discussed in reference 2.

Physically, the time constant can be considered as proportional to the product of three factors, functions of the wire material, size, and operating conditions, respectively:

$$M \sim \left(\frac{\rho_w s}{\sigma_w \alpha} \right) (d^4) \left(\frac{\bar{R} - R_a}{1^2 R_a} \right) \quad (6)$$

The attenuation and phase lag in the hot-wire response are compensated up to appropriate frequencies by a suitable network between two of the amplifier stages.

The technique for the measurement of specific statistical kinematic quantities in a turbulent flow need not be discussed in detail here. Adequate descriptions are given in other papers (references 2, 7, 18, and others). It is sufficient to point out that $\frac{u'}{\bar{U}}$ is computed directly from a measurement of $\sqrt{e^2}$ as given in equation (3). The components $\frac{v'}{\bar{U}}$, $\frac{w'}{\bar{U}}$, and $\frac{uv}{\bar{U}^2}$ are measured with directionally sensitive hot-wire instruments, customarily in the form of an X, with the polar axis perpendicular to the mean velocity direction. Each of the two wires in such an instrument, being inclined to the mean flow, responds to both longitudinal and lateral fluctuations, so that, instead of equation (3), there results

$$e_{1,2} = \delta \frac{u}{\bar{U}} \pm \epsilon \frac{v}{\bar{U}}$$

for the two wires. Thus, for example,

$$\frac{u'}{\bar{U}} \sim \sqrt{(e_1 + e_2)^2}$$

$$\frac{v'}{\bar{U}} \sim \sqrt{(e_1 - e_2)^2}$$

$$\frac{uv}{\bar{U}^2} \sim \overline{e_1^2} - \overline{e_2^2}$$

The correlation between u fluctuations at two different points is obtained in the same way as the shear. Integration of the correlation curve gives the scale of turbulence.

The energy spectrum of turbulence is obtained conventionally with a narrow band-pass filter.

Some corrections for the effect of wire length have been worked out in reference 7, but further detailed analysis along that line is necessary, particularly for the measurement of correlation functions and shear and turbulence spectrums. K. Zebb (reference 32) has made some approximate computations for the length correction of X meters in v' measurements.

MEASUREMENT OF MEAN VELOCITY AND TEMPERATURE

In applications of King's equation for thermal equilibrium

$$\frac{l^2 \bar{R}}{\bar{R} - \bar{R}_a} = \bar{A} + \bar{B} \sqrt{\bar{U}}$$

where

$$\bar{A} = \frac{al}{R_0 \alpha} \bar{k}$$

$$\bar{B} = \frac{bl}{R_0 \alpha} \sqrt{d \bar{c}_p \bar{\rho} \bar{k}}$$

to the measurement of mean velocity and temperature in flow with a temperature gradient, the only change from the first part of the preceding section is that $\bar{A}$ and $\bar{B}$ now vary from point to point in the flow. These variations are determined from the changes in the physical constants of the fluid. The present investigation is restricted to atmospheric air.

Since air is very nearly a perfect gas,

$$\frac{\rho}{\rho_r} = \frac{T_r}{T_a} \quad (7)$$

where the subscript r corresponds to an appropriate reference temperature.

From the International Critical Tables (reference 23), the thermal conductivity of air can be approximated for temperatures up to a few hundred degrees centigrade by the empirical equation

$$\frac{k}{k_r} = \frac{T_r + 125}{T_a + 125} \left(\frac{T_a}{T_r} \right)^{3/2} \quad (8)$$

This can be approximated by a straight line over a fairly wide temperature range, and, in fact, the linear approximation is used in the next section.

The variation in c_p with temperature can be neglected in the present discussion, from 0° C to about 300° C. Thus,

$$\frac{\bar{A}}{A_r} = \frac{\bar{k}}{k_r}$$

$$\frac{\bar{B}}{B_r} = \sqrt{\frac{\bar{k} \bar{\rho}}{k_r \rho_r}}$$

or

$$\left. \begin{aligned} \frac{\bar{A}}{A_r} &= \frac{T_r + 125}{T_a + 125} \left(\frac{T_a}{T_r} \right)^{3/2} \\ \frac{\bar{B}}{B_r} &= \sqrt{\frac{T_r + 125}{T_a + 125} \left(\frac{T_a}{T_r} \right)^{1/2}} \end{aligned} \right\} \quad (9)$$

In practice, it is convenient to have these expressions in terms of wire resistances. With

$$\left. \begin{aligned} R_a &= R_0 [1 + \alpha(T_a - 273)] \\ R_r &= R_0 [1 + \alpha(T_r - 273)] \end{aligned} \right\} \quad (10)$$

equation (9) can be written in the form,

$$\left. \begin{aligned} \frac{\bar{A}}{A_r} &= \frac{T_r + 125}{T_r + 125 + \left[\frac{1}{\alpha} + (T_r - 273) \left(\frac{\bar{R}_a}{R_r} - 1 \right) \right]} \left\{ \frac{T_r + \left[\frac{1}{\alpha} + (T_r - 273) \right] \left(\frac{\bar{R}_a}{R_r} - 1 \right)}{T_r} \right\}^{3/2} \\ \frac{\bar{B}}{B_r} &= \left\{ \frac{T_r + 125}{T_r + 125 + \left[\frac{1}{\alpha} + (T_r - 273) \right] \left(\frac{\bar{R}_a}{R_r} - 1 \right)} \right\}^{1/2} \left\{ \frac{T_r + \left[\frac{1}{\alpha} + (T_r - 273) \right] \left(\frac{\bar{R}_a}{R_r} - 1 \right)}{T_r} \right\}^{1/4} \end{aligned} \right\} \quad (11)$$

For a platinum wire, $\alpha = 0.0037$ and $\frac{1}{\alpha} = 270$, and these can be approximated by

$$\left. \begin{aligned} \frac{\bar{A}}{A_r} &= \left(\frac{T_r + 125}{\frac{\bar{R}_a}{R_r} T_r + 125} \right) \left(\frac{\bar{R}_a}{R_r} \right)^{3/2} \\ \frac{\bar{B}}{B_r} &= \left(\frac{T_r + 125}{\frac{\bar{R}_a}{R_r} T_r + 125} \right)^{1/2} \left(\frac{\bar{R}_a}{R_r} \right)^{1/4} \end{aligned} \right\} \quad (12)$$

These are presented in figure 1 for $T_r = 293^\circ$ abs.

With these values of $\bar{A}$ and $\bar{B}$, the mean velocity is determined just as in the case of the simple anemometer. The mean temperature $\bar{T}_a$ is, of course, obtained directly from equation (10) which can be combined to give

$$\bar{T}_a = T_r + \left[\frac{1}{\alpha} + (T_r - 273) \right] \left(\frac{\bar{R}_a}{R_r} - 1 \right) \quad (13)$$

which, for a platinum wire becomes approximately

$$\bar{T}_a = T_r \left(\frac{\bar{R}_a}{R_r} \right)$$

It should be emphasized that the accuracy of these results, as well as all those in the following sections, depends upon the accuracy of the physical form of the cooling terms in King's equations. The linear

variation of $\frac{i^2 \bar{R}}{\bar{R} - \bar{R}_a}$ with $\bar{U}^{1/2}$ has been checked with considerable

accuracy by many experimenters; figure 2 is a typical isothermal mean-velocity calibration of a hot-wire anemometer. However, there has apparently been no attempt to check the exponents of the other physical quantities as derived by King. Appendix A contains a discussion of this problem, as well as some preliminary experimental results.

RESPONSE OF A HOT-WIRE IN AN AIR STREAM WITH VELOCITY AND TEMPERATURE FLUCTUATIONS

Voltage Fluctuation

In order to deduce an equation analogous to equation (3), start with King's equation for thermal equilibrium,

$$\frac{i^2 R}{R - R_a} = \frac{a l}{R_0 \alpha} k + \frac{b l}{R_0 \alpha} (dc_p \rho k U)^{1/2}$$

With small fluctuations in velocity and air temperature, there are introduced

$$\begin{aligned} R &= \bar{R} + r & U &= \bar{U} + u \\ R_a &= \bar{R}_a + r_a & \rho &= \bar{\rho} + \rho' \\ k &= \bar{k} + k' & c_p &= \bar{c}_p \end{aligned}$$

so that King's equation becomes, writing $\frac{a l}{R_0 \alpha} \equiv P$ and $\frac{b l \sqrt{d c_p}}{R_0 \alpha} \equiv Q$,

$$\frac{i^2 (\bar{R} + r)}{(\bar{R} - \bar{R}_a) + (r - r_a)} = P(\bar{k} + k') + Q [(\bar{\rho} + \rho')(\bar{k} + k')(\bar{U} + u)]^{1/2}$$

Using a binomial expansion for the denominator of the left side and neglecting products of small quantities on both sides,

$$\begin{aligned} \frac{i^2 \bar{R}}{\bar{R} - \bar{R}_a} + \frac{i^2}{(\bar{R} - \bar{R}_a)^2} (\bar{R} r_a - \bar{R}_a r) &= P(\bar{k} + k') \\ &+ Q(\bar{\rho} \bar{k} \bar{U} + \rho' \bar{k} \bar{U} + \bar{\rho} k' \bar{U} + \bar{\rho} \bar{k} u)^{1/2} \end{aligned}$$

Keeping the linear terms of the binomial expansion of the last part,

$$\begin{aligned} \frac{i^2 \bar{R}}{\bar{R} - \bar{R}_a} + \frac{i^2}{(\bar{R} - \bar{R}_a)^2} (\bar{R} r_a - \bar{R}_a r) &= P(\bar{k} + k') \\ &+ Q(\bar{\rho} \bar{k} \bar{U})^{1/2} \left[1 + \frac{1}{2} \left(\frac{\rho'}{\bar{\rho}} + \frac{k'}{\bar{k}} + \frac{u}{\bar{U}} \right) \right] \end{aligned}$$

In order to get the fluctuation equation, subtract the averaged King's equation,

$$\frac{i^2 \bar{R}}{\bar{R} - \bar{R}_a} = P \bar{k} + Q(\bar{\rho} \bar{k} \bar{U})^{1/2}$$

which leaves

$$\frac{i^2}{(\bar{R} - \bar{R}_a)^2} (\bar{R} r_a - \bar{R}_a r) = Pk' + \frac{Q}{2} (\rho k U)^{1/2} \left(\frac{\rho'}{\bar{\rho}} + \frac{k'}{\bar{k}} + \frac{u}{\bar{U}} \right) \quad (14)$$

and the next step is to express r_a , k' , and ρ' in terms of the small temperature fluctuation ϑ .

Obviously,

$$r_a = R_0 \alpha \vartheta \quad (15)$$

As mentioned in the previous section, the rather complicated empirical expression for the temperature dependence of the thermal conductivity of air (equation (8)) can be well approximated over a large temperature range by a single straight line. Hence, with considerable accuracy, there can be written

$$k' = n \vartheta \quad (16)$$

where n is sensibly constant over a range of a few hundred degrees centigrade.

For the density fluctuation, again air is considered as a perfect gas and the $\rho(T)$ hyperbola in the vicinity of $\bar{T}_a$ is approximated by its tangent. Thus,

$$\rho = \bar{\rho} \frac{\bar{T}_a}{T_a} = \bar{\rho} + \rho'$$

Therefore,

$$\bar{\rho} + \rho' = \bar{\rho} \frac{\bar{T}_a}{\bar{T}_a + \vartheta} = \bar{\rho} \frac{1}{1 + \frac{\vartheta}{\bar{T}_a}}$$

and for $\frac{\vartheta}{\bar{T}_a} \ll 1$, there results

$$\rho' = -\bar{\rho} \frac{\vartheta}{\bar{T}_a} \quad (17)$$

Substituting equations (15), (16), and (17) and introducing the hot-wire voltage fluctuation $e = ir$, as well as the expressions for P and Q ,

$$\begin{aligned} & \frac{i^2 \bar{R} R_0 \alpha}{(\bar{R} - \bar{R}_a)^2} \vartheta - \frac{i \bar{R}_a}{(\bar{R} - \bar{R}_a)^2} e \\ &= \frac{a \ln \vartheta}{R_0 \alpha} + \frac{b l}{R_0 \alpha} (\text{dc}_p \bar{\rho} \bar{k} \bar{U})^{1/2} \left(-\frac{\vartheta}{\bar{T}_a} + \frac{n \vartheta}{k} + \frac{u}{\bar{U}} \right) \end{aligned} \quad (18)$$

Calculation with typical numerical values (see appendix B) shows that, for most practical purposes, the sum of the two ϑ -terms in the final parenthesis of equation (18) can be neglected in comparison with the first term in the equation, so they will be omitted in the rest of the discussion.

Collecting the remaining terms,

$$\frac{i \bar{R}_a}{(\bar{R} - \bar{R}_a)^2} e = \left[\frac{i^2 \bar{R} R_0 \alpha}{(\bar{R} - \bar{R}_a)^2} - \frac{a \ln \vartheta}{R_0 \alpha} \right] \vartheta + \frac{b l}{2 R_0 \alpha} (\text{dc}_p \bar{\rho} \bar{k} \bar{U})^{1/2} \frac{u}{\bar{U}}$$

The functional form desired is $e = e\left(\frac{\vartheta}{\bar{\theta}}, \frac{u}{\bar{U}}\right)$. Since

$$\bar{\theta} = \bar{T}_a - T_r = \frac{\bar{R}_a - R_r}{R_0 \alpha}$$

the ϑ -term is multiplied by $\left(\frac{\bar{R}_a - R_r}{R_0 \alpha} \frac{1}{\bar{\theta}}\right)$. It is also convenient to

replace $\frac{b l}{R_0 \alpha} (\text{dc}_p \bar{\rho} \bar{k} \bar{U})^{1/2}$ by its equivalent as given in King's equation.

Then, finally,

$$e = \frac{\bar{R}_a - R_r}{i \bar{R}_a} \left[i^2 \bar{R} - a \ln \frac{(\bar{R} - \bar{R}_a)^2}{(R_0 \alpha)^2} \right] \frac{\vartheta}{\bar{\theta}} - \frac{\bar{R} - \bar{R}_a}{2 i \bar{R}_a} \left[i^2 \bar{R} - \bar{A}(\bar{R} - \bar{R}_a) \right] \frac{u}{\bar{U}} \quad (19)$$

which obviously reduces to equation (3) when the temperature fluctuation level becomes much smaller than the velocity fluctuation level. The value

of $\bar{A}$ is determined from the mean-speed calibration of the hot wire, and, of course, $\frac{a l}{R_0 \alpha} = \frac{\bar{A}}{k}$.

Time Constant

The general equation is given under CONVENTIONAL HOT-WIRE-ANEMOMETER RESPONSE EQUATIONS as follows:

$$\frac{4.2ms}{R_0 \alpha} \frac{dR}{dt} = i^2 R - (A + B \sqrt{U})(R - R_a) \quad (1a)$$

Equation (2) describes a hypothetical equilibrium reached at the instantaneous flow conditions. Substituting for $(A + B \sqrt{U})$ from equation (2), there is obtained

$$\frac{4.2ms}{R_0 \alpha} \frac{dR}{dt} = \frac{i^2 R_a (R_e - R)}{R_e - R_a} \quad (20)$$

which is the equivalent of equation (4) of reference 2, with the essential difference that here R_a is a fluctuating function of time.

Since, as for the simple anemometer, the general solution of equation (20) is apparently not expressible in any applicable form, possible simplifications are now considered for the restriction of small fluctuations, that is,

$$\left| \frac{R - \bar{R}}{\bar{R}} \right| \ll 1 \quad \text{and} \quad \left| \frac{R_a - \bar{R}_a}{\bar{R}_a} \right| \ll 1$$

Hence R_a may be replaced by $\bar{R}_a$ in the numerator of the right side of equation (20). The denominator may be examined in somewhat the same way as Dryden and Kueth obtained their equation (15); that is, there may be written

$$R_e - R_a = (\bar{R} - \bar{R}_a) + (R_e - \bar{R}) + (\bar{R}_a - R_a)$$

Then it is seen that the last two parentheses are small compared with the first if appreciable heating current is used. In some cases, when only the temperature fluctuation level is being measured, relatively little heating current is used. With negligible sensitivity to velocity, it is

apparent that $R_e - R_a \doteq \text{Constant}$ and, therefore, may be replaced by the average $\bar{R}_e - \bar{R}_a$. However, for small fluctuations, $\bar{R}_e \doteq \bar{R}$, so that $R_e - R_a$ may be replaced by $\bar{R} - \bar{R}_a$. Equation (20) becomes approximately,

$$\frac{4.2ms}{R_0\alpha} \frac{dR}{dt} = \frac{1^2 \bar{R}_a (R_e - R)}{\bar{R} - \bar{R}_a} \quad (21)$$

or

$$M' \frac{dR}{dt} = R_e - R \quad (21a)$$

where

$$M' = \frac{4.2ms(\bar{R} - \bar{R}_a)}{1^2 \bar{R}_a R_0 \alpha} \quad (22)$$

so that equation (21a) is identical with Dryden and Kuethe's equation (15), and $M' = M$, the simple anemometric time constant. Clearly, from the foregoing analysis, this will not be true for large fluctuations.

With negligible heating current, equation (1a) becomes

$$\frac{4.2ms}{R_0\alpha} \frac{dR}{dt} = -(A + B\sqrt{U})(R - R_a)$$

From equilibrium, $R_e = R_a$. Therefore,

$$\frac{4.2ms}{R_0\alpha} \frac{dR}{dt} = (A + B\sqrt{U})(R_e - R)$$

or

$$M' \frac{dR}{dt} = R_e - R \quad (21a)$$

where

$$M' = \frac{4.2ms}{R_0\alpha(\bar{A} + \bar{B}\sqrt{\bar{U}})} \quad (23)$$

But

$$(\bar{A} + \bar{B} \sqrt{\bar{U}}) = \frac{i^2 \bar{R}}{\bar{R} - \bar{R}_a} = \frac{i^2 \bar{R}_a}{\bar{R} - \bar{R}_a}$$

for the case of extremely small current, so that equation (23) is effectively the same as equation (22). A more direct connection appears if equation (22) is written as

$$M' = \frac{4.2 \text{ ms}}{R_0 \alpha} \frac{1}{(\bar{A} - i^2) + \bar{B} \sqrt{\bar{U}}} \quad (22a)$$

(See appendix C.)

MEASUREMENT PROCEDURES FOR VELOCITY

AND TEMPERATURE FLUCTUATIONS

Single Normal Wire

It has been seen that the response of a hot-wire to simultaneous velocity and temperature fluctuations is given approximately by

$$e = \frac{\bar{R}_a - R_r}{i \bar{R}_a} \left[i^2 \bar{R} - a \ln \frac{(\bar{R} - \bar{R}_a)^2}{(R_0 \alpha)^2} \right] \frac{\partial}{\partial \theta} - \frac{\bar{R} - \bar{R}_a}{2 i \bar{R}_a} \left[i^2 \bar{R} - \bar{A}(\bar{R} - \bar{R}_a) \right] \frac{u}{\bar{U}} \quad (19)$$

This can be written simply as

$$e = \beta \frac{\partial}{\partial \theta} + \delta \frac{u}{\bar{U}} \quad (19a)$$

where β and δ are functions of the mean-velocity calibration constants and of the hot-wire operating conditions.

First consider the quantities that can be obtained from the root-mean-square output of single wires set perpendicular to the mean flow:

$$\overline{e^2} = \beta^2 \overline{\left(\frac{\partial}{\partial \theta} \right)^2} + 2\beta\delta \overline{\left(\frac{\partial u}{\partial \theta \bar{U}} \right)} + \delta^2 \overline{\left(\frac{u}{\bar{U}} \right)^2} \quad (24)$$

Since β and δ do not increase in the same proportion for a given increase in hot-wire temperature, readings of e^2 at three different currents at the same point in the flow will give simultaneous linear algebraic equations from which the three unknown quantities can be obtained:

$$\left. \begin{aligned} \overline{e_1^2} &= \beta_1^2 \left(\frac{\partial}{\partial \theta} \right)^2 + 2\beta_1\delta_1 \left(\frac{\partial u}{\partial \theta \partial U} \right) + \delta_1^2 \left(\frac{u}{U} \right)^2 \\ \overline{e_2^2} &= \beta_2^2 \left(\frac{\partial}{\partial \theta} \right)^2 + 2\beta_2\delta_2 \left(\frac{\partial u}{\partial \theta \partial U} \right) + \delta_2^2 \left(\frac{u}{U} \right)^2 \\ \overline{e_3^2} &= \beta_3^2 \left(\frac{\partial}{\partial \theta} \right)^2 + 2\beta_3\delta_3 \left(\frac{\partial u}{\partial \theta \partial U} \right) + \delta_3^2 \left(\frac{u}{U} \right)^2 \end{aligned} \right\} \quad (25)$$

It immediately follows that

$$\left(\frac{\partial}{\partial \theta} \right)^2 = \frac{1}{\Delta} \begin{vmatrix} \overline{e_1^2} & 2\beta_1\delta_1 & \delta_1^2 \\ \overline{e_2^2} & 2\beta_2\delta_2 & \delta_2^2 \\ \overline{e_3^2} & 2\beta_3\delta_3 & \delta_3^2 \end{vmatrix}$$

$$\left(\frac{\partial u}{\partial \theta \partial U} \right) = \frac{1}{\Delta} \begin{vmatrix} \beta_1^2 & \overline{e_1^2} & \delta_1^2 \\ \beta_2^2 & \overline{e_2^2} & \delta_2^2 \\ \beta_3^2 & \overline{e_3^2} & \delta_3^2 \end{vmatrix}$$

$$\left(\frac{u}{U} \right)^2 = \frac{1}{\Delta} \begin{vmatrix} \beta_1^2 & 2\beta_1\delta_1 & \overline{e_1^2} \\ \beta_2^2 & 2\beta_2\delta_2 & \overline{e_2^2} \\ \beta_3^2 & 2\beta_3\delta_3 & \overline{e_3^2} \end{vmatrix}$$

where

$$\Delta = \begin{vmatrix} \beta_1^2 & 2\beta_1\delta_1 & \delta_1^2 \\ \beta_2^2 & 2\beta_2\delta_2 & \delta_2^2 \\ \beta_3^2 & 2\beta_3\delta_3 & \delta_3^2 \end{vmatrix}$$

In general, because of the rather complicated forms of β and δ , these determinants will be tedious to evaluate. However, the procedure may be made a bit more convenient by fixing the ratio $\bar{R}_1:\bar{R}_2:\bar{R}_3$ and setting up appropriate calculation charts.

Possible Short Cuts

In order to measure either $\left(\frac{\theta}{\bar{\theta}}\right)^2$ or $\left(\frac{u}{\bar{U}}\right)^2$ directly, it is necessary to make either δ or β negligible; that is,

$$(\bar{R}_a - R_r) \left[1^2 \bar{R} - a \ln \frac{(\bar{R} - \bar{R}_a)^2}{(R_0 \alpha)^2} \right] \begin{matrix} >> \\ \text{or} \\ << \end{matrix} \frac{1}{2} (\bar{R} - \bar{R}_a) \left[1^2 \bar{R} - \bar{A} (\bar{R} - \bar{R}_a) \right]$$

The meaning of this can be found in terms of operating conditions at a given point in the flow by replacing the electrical quantities with fluid mechanical quantities as far as possible. Then the inequality can be written, after some algebra, as

$$\left[\left(\bar{A} + \bar{B} \sqrt{\bar{U}} \right) \frac{R_0 \alpha}{(\bar{R} - \bar{R}_a)} - \frac{\bar{A}}{\bar{K}} \right] \bar{\theta} \begin{matrix} >> \\ \text{or} \\ << \end{matrix} \frac{\bar{B}}{2} \sqrt{\bar{U}} \quad (26)$$

For the entire range of practicable operation, the first term in the parenthesis in equation (26) is larger than the second. Therefore, it is evident that, for a given flow, β is maximized relative to δ by minimizing $\bar{R} - \bar{R}_a$, and the converse is also true. Whether either of these extremes can, in practice, make $\beta \gg \delta$ or $\beta \ll \delta$ can be checked numerically from a typical hot-wire calibration. It will be noted that the first term in the parenthesis is the only one of the three that can be varied by changing the hot-wire current. It turns out that, when both $\bar{\theta}$ and $\bar{U}$ are large, the only feasible inequality is that obtained by reducing the heating current so that $\beta \gg \delta$; this ordinarily leads to the additional simplification,

$$\beta \doteq 1(\bar{R}_a - R_r)$$

for the sensitivity of the simple resistance thermometer.

Equation (26) also gives the obvious result that reducing $\bar{\theta}$ toward zero will eventually make $\beta \ll \delta$, and then the wire is a simple anemometer.

Another possibility for direct measurement is the use of two close parallel wires of different diameter, the voltage difference being the signal to the amplifier. Then, if $\beta_1 = \beta_2$, the difference will

measure $\left(\frac{u}{U}\right)^2$, while, if $\delta_1 = \delta_2$, the difference will measure $\left(\frac{\vartheta}{\theta}\right)^2$.

Of course, they must be far enough apart to be insensitive to lateral velocity fluctuations.

The general requirements for $\beta_1 = \beta_2$ are extremely complicated if the general expression for β is used. However, for fairly small currents, the second term in β is negligible compared with the first. In cases where this is true, $\beta_1 = \beta_2$ means

$$i_1 \frac{\bar{R}_1}{\bar{R}_{a1}} (\bar{R}_{a1} - R_{r1}) = i_2 \frac{\bar{R}_2}{\bar{R}_{a2}} (\bar{R}_{a2} - R_{r2})$$

But

$$\frac{\bar{R}_{a1} - R_{r1}}{\bar{R}_{a1}} = \frac{\bar{R}_{a2} - R_{r2}}{\bar{R}_{a2}}$$

so that

$$i_1 \bar{R}_1 = i_2 \bar{R}_2 \quad (27)$$

This relation can be satisfied physically, but the problem of correct compensation for the two wires of different diameters would probably require that the fluctuating voltages be subtracted after leaving matched amplifiers, before entering the output circuit.

The requirement for $\delta_1 = \delta_2$ is rather complicated, and this case will not ordinarily be necessary, since it is usually possible to make $\delta \ll \beta$.

Directionally Sensitive Meters (e.g., X and ||)

In an ideal meter, the two wires are identical, so that the voltage fluctuations will be

$$\left. \begin{aligned} e_1 &= \beta \frac{\vartheta}{\theta} + \delta \frac{u}{U} + \epsilon \frac{v}{U} \\ e_2 &= \beta \frac{\vartheta}{\theta} + \delta \frac{u}{U} - \epsilon \frac{v}{U} \end{aligned} \right\} \quad (28)$$

Subtracting the voltages before amplification and measuring the mean square leads to

$$\overline{\left(\frac{v}{U}\right)^2} = \frac{(e_1 - e_2)^2}{4\epsilon^2} \quad (29)$$

thus giving the lateral turbulence level.

Taking the mean square output of each wire separately, there results

$$\overline{e_1^2} = \beta^2 \overline{\left(\frac{\partial}{\partial \theta}\right)^2} + \delta^2 \overline{\left(\frac{u}{U}\right)^2} + \epsilon^2 \overline{\left(\frac{v}{U}\right)^2} + 2\beta\delta \overline{\frac{\partial u}{\partial U}} + 2\beta\epsilon \overline{\frac{\partial v}{\partial U}} + 2\delta\epsilon \overline{\frac{uv}{U^2}}$$

$$\overline{e_2^2} = \beta^2 \overline{\left(\frac{\partial}{\partial \theta}\right)^2} + \delta^2 \overline{\left(\frac{u}{U}\right)^2} + \epsilon^2 \overline{\left(\frac{v}{U}\right)^2} + 2\beta\delta \overline{\frac{\partial u}{\partial U}} - 2\beta\epsilon \overline{\frac{\partial v}{\partial U}} - 2\delta\epsilon \overline{\frac{uv}{U^2}}$$

After subtraction,

$$\overline{e_1^2} - \overline{e_2^2} = 4\beta\epsilon \overline{\frac{\partial v}{\partial U}} + 4\delta\epsilon \overline{\frac{uv}{U^2}} \quad (30)$$

The turbulent momentum and heat transfer are determined by repeating equation (30) at a different current and solving the resulting pair of simultaneous linear equations. Of course, the two terms in equation (30) are proportional to the turbulent shear and heat transfer only if the density fluctuations can be neglected (appendix D).

If the hot-wire instrument is unsymmetrical, the analysis is more complicated but essentially the same. For purely kinematic measurements, the generalization is given in reference 19.

Just as in the case of the single wire, direct measurement of $\overline{\frac{\partial v}{\partial U}}$ or $\overline{\frac{uv}{U^2}}$ is possible if $\delta \ll \beta$ or $\delta \gg \beta$, respectively. Again it is to be expected that in a fairly hot jet only the former condition is obtainable.

In the general case, corresponding to equation (30), and in all other correlation measurements involving thermal and kinematic variables simultaneously, the direct measurement of correlation coefficients

(e.g., $\frac{\overline{v}}{v'}$, $\frac{\overline{uv}}{u'v'}$, etc.) is apparently not usually possible. Such direct determination is common practice in purely kinematic measurements.

Correlations between Different Points

For two single wires set parallel to each other and perpendicular to the flow, assuming the ideal case of identical wires at the same sensitivity, the following equations apply:

$$\left. \begin{aligned} e_1 &= \beta \frac{v_1}{\theta} + \delta \frac{u_1}{U} \\ e_2 &= \beta \frac{v_2}{\theta} + \delta \frac{u_2}{U} \end{aligned} \right\} \quad (31)$$

Adding and subtracting the fluctuating voltages gives

$$\begin{aligned} \overline{(e_1 + e_2)^2} &= \beta^2 \overline{\left(\frac{v_1 + v_2}{\theta}\right)^2} + 2\beta\delta \overline{\left(\frac{v_1 + v_2}{\theta}\right)\left(\frac{u_1 + u_2}{U}\right)} + \delta^2 \overline{\left(\frac{u_1 + u_2}{U}\right)^2} \\ \overline{(e_1 - e_2)^2} &= \beta^2 \overline{\left(\frac{v_1 - v_2}{\theta}\right)^2} + 2\beta\delta \overline{\left(\frac{v_1 - v_2}{\theta}\right)\left(\frac{u_1 - u_2}{U}\right)} + \delta^2 \overline{\left(\frac{u_1 - u_2}{U}\right)^2} \end{aligned}$$

Expanding the foregoing expressions and subtracting gives

$$\overline{(e_1 + e_2)^2} - \overline{(e_1 - e_2)^2} = 4\beta^2 \overline{\left(\frac{v_1 v_2}{\theta^2}\right)} + 4\beta\delta \left(\overline{\frac{v_1 u_2}{\theta U}} + \overline{\frac{v_2 u_1}{\theta U}} \right) + 4\delta^2 \overline{\left(\frac{u_1 u_2}{U^2}\right)} \quad (32)$$

and the three quantities in parentheses on the right side can be obtained by making readings at three different hot-wire currents. The physical significance of the cross-product term is not immediately evident.

Of course, with $\beta \gg \delta$, the thermal correlation function, and hence the thermal scale, is measurable directly.

The correlation between v fluctuations at different points can be determined by using two double wire meters, measuring the mean square of the sum of the differences of the individual meter differences. It can also be measured with symmetrically inclined single wires.

Spectrum

If $\frac{\vartheta'}{\bar{\theta}}$ and $\frac{u'}{\bar{U}}$ are the same order of magnitude in a flow with appreciable velocity and temperature differences, it may be possible to measure only the ϑ' spectrum directly. This is true for cases in which it is not feasible to make $\delta \gg \beta$ because of wire temperature limitations. Then it may be necessary to obtain the u' spectrum with a meter made up of two wires of differing diameter as described previously.

For temperature differences up to perhaps 100° C, however, it does not seem likely that the nature of the turbulence will be changed essentially in a flow of moderate velocity. Therefore, to a good approximation, it would seem satisfactory to determine the u' spectrum at reduced $\bar{\theta}$.

It should be noted that the v' spectrum is directly measurable in all cases.

MEASUREMENT OF MEAN VELOCITY AND CONCENTRATION

IN AN ISOTHERMAL FLOWING MIXTURE OF TWO GASES

As in the section dealing with the measurement of mean velocity and temperature, start with King's equation for thermal equilibrium,

$$\frac{1^2 \bar{R}}{\bar{R} - R_a} = \bar{A} + \bar{B} \sqrt{\bar{U}}$$

where

$$\bar{A} = \frac{al}{R_0 \alpha} \bar{k}$$

and

$$\bar{B} = \frac{bl}{R_0 \alpha} (\bar{d} \bar{c}_p \bar{\rho} \bar{k} \bar{U})^{1/2}$$

Again, $\bar{A}$ and $\bar{B}$ are not constants but depend upon the local physical constants of the flow and, therefore, are a function of the relative concentration.

As indicated in the symbols, the concentration used in this analysis is the molal concentration. Further remarks on this are included in appendix E.

The expressions for the mean physical constants of the mixture are written as

$$\left. \begin{aligned} \bar{\rho} &= \rho_a + \bar{\Gamma} \Delta\rho \\ \bar{c}_p &= c_{p_a} + \bar{\Gamma} \Delta c_p \\ \bar{k} &= k_a + f(\bar{\Gamma}) \Delta k \end{aligned} \right\} \quad (33)$$

The thermal conductivity of gas mixtures is generally not a simple weighted average (reference 24) and varies considerably for different pairs of gases. If $k(\Gamma)$ is a known function, the procedure is relatively simple, and, for most pairs of gases for which data are available (reference 25), $f(\Gamma)$ can be well approximated by a quadratic or cubic polynomial. If $k(\Gamma)$ is unknown, which is still the case for many pairs of common gases, it can be determined as one of the unknowns by an extra measurement at each point in the flow. Both cases are treated here.

$k(\Gamma)$ a Known Function

For the discussion of $k(\Gamma)$ as a known function, assume that $f(\Gamma)$ can be approximated by a parabola:

$$\bar{k} = k_a + (\xi \bar{\Gamma} + \zeta \bar{\Gamma}^2) \Delta k \quad (33a)$$

Equation (2) can be written as

$$\begin{aligned} \frac{i^2 \bar{R}}{\bar{R} - R_a} &= \frac{a l}{R_0 \alpha} \left[k_a + (\xi \bar{\Gamma} + \zeta \bar{\Gamma}^2) \Delta k \right] \\ &+ \frac{b l \sqrt{d}}{R_0 \alpha} \left(c_{p_a} + \bar{\Gamma} \Delta c_p \right)^{1/2} \left(\rho_a + \bar{\Gamma} \Delta \rho \right)^{1/2} \left[k_a + (\xi \bar{\Gamma} + \zeta \bar{\Gamma}^2) \Delta k \right]^{1/2} \bar{U}^{1/2} \quad (34) \end{aligned}$$

Equation (34) is to be used to transform appropriate readings of $\frac{i^2 \bar{R}}{\bar{R} - R_a}$ at a point into values of the two unknowns $\bar{U}$ and $\bar{\Gamma}$. Perhaps the most convenient method is to record $\frac{i^2 \bar{R}}{\bar{R} - R_a}$ at the same point with two hot-wires differing in diameter, so that $\left(\frac{b l \sqrt{d}}{R_0 \alpha}\right)_1 \neq \left(\frac{b l \sqrt{d}}{R_0 \alpha}\right)_2$. According to King's equation, the coefficients of the first term will be the same if the wires are of the same material, but, experimentally, they usually turn out to be different also.

Then the readings of the two hot-wires yield simultaneous equations for $\bar{U}$ and $\bar{\Gamma}$:

$$\phi_1 = M_1 \left[k_a + (\xi \bar{\Gamma} + \zeta \bar{\Gamma}^2) \Delta k \right] + N_1 \left(c_{p_a} + \bar{\Gamma} \Delta c_p \right)^{1/2} (\rho_a + \bar{\Gamma} \Delta \rho)^{1/2} \left[k_a + (\xi \bar{\Gamma} + \zeta \bar{\Gamma}^2) \Delta k \right]^{1/2} \bar{U}^{1/2}$$

$$\phi_2 = M_2 \left[k_a + (\xi \bar{\Gamma} + \zeta \bar{\Gamma}^2) \Delta k \right] + N_2 \left(c_{p_a} + \bar{\Gamma} \Delta c_p \right)^{1/2} (\rho_a + \bar{\Gamma} \Delta \rho)^{1/2} \left[k_a + (\xi \bar{\Gamma} + \zeta \bar{\Gamma}^2) \Delta k \right]^{1/2} \bar{U}^{1/2}$$

where

$$\phi = \frac{i^2 \bar{R}}{\bar{R} - R_a}$$

$$M = \frac{a l}{R_0 \alpha}$$

$$N = \frac{b l \sqrt{d}}{R_0 \alpha}$$

Multiplying the first equation by N_2 and the second by N_1 and subtracting, a solution can be obtained for $\left[k_a + (\xi\bar{\Gamma} + \zeta\bar{\Gamma}^2)\Delta k \right]$, and the resulting quadratic equation can be solved for $\bar{\Gamma}$. Provided that $k(\Gamma)$ is a monotonic function, only one of the two solutions of this quadratic will have any physical meaning. This value of $\bar{\Gamma}$ can then be substituted into either equation to obtain $\bar{U}$. This latter step can be done graphically if a mean-velocity calibration is drawn up of either hot wire; that

is, $\frac{1^2\bar{R}}{\bar{R} - R_a}$ against $\sqrt{\bar{U}}$ with straight lines of constant $\bar{\Gamma}$. The two limiting lines $\bar{\Gamma} = 0$ and $\bar{\Gamma} = 1.0$ can be determined experimentally.

For research in turbulent material diffusion it will be of particular interest to use a gas such that $\rho_g = \rho_a$ (e.g., ethylene). In that case, a somewhat shorter technique can be used: A pitot tube will give $\bar{U}$ directly and then $\bar{\Gamma}$ can be determined from a single hot-wire reading and the family of calibration curves just described.

$k(\Gamma)$ an Unknown Function

In the case of $k(\Gamma)$ as an unknown function, it is convenient to work with the readings of two different hot-wires and a pitot tube at each point.

The two hot-wires give

$$\left. \begin{aligned} \phi_1 &= M_1 \left[k_a + f(\bar{\Gamma})\Delta k \right] + N_1 (c_{p_a} + \bar{\Gamma} \Delta c_p)^{1/2} (\rho_a + \bar{\Gamma} \Delta \rho)^{1/2} \left[k_a + f(\bar{\Gamma})\Delta k \right]^{1/2} \bar{U}^{1/2} \\ \phi_2 &= M_2 \left[k_a + f(\bar{\Gamma})\Delta k \right] + N_2 (c_{p_a} + \bar{\Gamma} \Delta c_p)^{1/2} (\rho_a + \bar{\Gamma} \Delta \rho)^{1/2} \left[k_a + f(\bar{\Gamma})\Delta k \right]^{1/2} \bar{U}^{1/2} \end{aligned} \right\} \quad (35)$$

where

$$\phi = \frac{1^2\bar{R}}{\bar{R} - R_a}$$

$$M = \frac{al}{R_0\alpha}$$

$$N = \frac{bl\sqrt{d}}{R_0\alpha}$$

Multiplying the first equation by N_2 and the second by N_1 and subtracting yields

$$k_a + f(\bar{\Gamma})\Delta k = \frac{\phi_1 N_2 - \phi_2 N_1}{M_1 N_2 - M_2 N_1} \equiv \kappa_a \quad (36)$$

Substituting equation (36) into the expression for ϕ_1 ,

$$\phi_1 = M_1 \kappa_a + N_1 (c_p + \bar{\Gamma} \Delta c_p)^{1/2} (\rho_a + \bar{\Gamma} \Delta \rho)^{1/2} \kappa_a^{1/2} \bar{U}^{1/2}$$

or

$$(c_{p_a} + \bar{\Gamma} \Delta c_p)^{1/2} (\rho_a + \bar{\Gamma} \Delta \rho)^{1/2} \bar{U}^{1/2} = \frac{\phi_1 - M_1 \kappa_a}{N_1 \kappa_a^{1/2}} \equiv \kappa_b \quad (37)$$

A reading at the same point with a pitot tube gives

$$q = \frac{1}{2} (\rho_a + \bar{\Gamma} \Delta \rho) \bar{U}^2 \quad (38)$$

Thus,

$$(\rho_a + \bar{\Gamma} \Delta \rho)^{1/2} = \frac{(2q)^{1/2}}{\bar{U}}$$

which is substituted into equation (37) to give $\bar{\Gamma}(\bar{U})$:

$$\bar{\Gamma} = \frac{1}{\Delta c_p} \left(\frac{\kappa_b^2}{2q} \bar{U} - c_{p_a} \right) \quad (39)$$

However, equation (38) also gives $\bar{\Gamma}(\bar{U})$ directly:

$$\bar{\Gamma} = \frac{1}{\Delta \rho} \left(\frac{2q}{\bar{U}^2} - \rho_a \right) \quad (38a)$$

And these two expressions are equated to give a cubic for $\bar{U}$:

$$\bar{U}^3 + \frac{2q}{\kappa_b^2} \frac{\Delta c_p}{\Delta \rho} \left(\frac{\rho_a}{\Delta c_p} - \frac{c_{pa}}{\Delta c_p} \right) \bar{U}^2 - \frac{4q^2}{\kappa_b^2} \frac{\Delta c_p}{\Delta \rho} = 0 \quad (40)$$

With the value of $\bar{U}$ at a point, $\bar{\Gamma}$ is obtained from equation (38a) or equation (39). The last unknown is the function $\bar{k}(\bar{\Gamma})$.

The values of κ_a at several points immediately permit solution for $f(\bar{\Gamma})$. For example, suppose it is assumed that f can be approximated by a cubic $f = a_1 \bar{\Gamma} + a_2 \bar{\Gamma}^2 + a_3 \bar{\Gamma}^3$. From readings at three different points in the flow, equation (36) yields three simultaneous equations for the unknown coefficients,

$$\left. \begin{aligned} \bar{\Gamma}_1 a_1 + \bar{\Gamma}_1^2 a_2 + \bar{\Gamma}_1^3 a_3 &= \frac{\kappa_{a1} - k_a}{\Delta k} \\ \bar{\Gamma}_2 a_1 + \bar{\Gamma}_2^2 a_2 + \bar{\Gamma}_2^3 a_3 &= \frac{\kappa_{a2} - k_a}{\Delta k} \\ \bar{\Gamma}_3 a_1 + \bar{\Gamma}_3^2 a_2 + \bar{\Gamma}_3^3 a_3 &= \frac{\kappa_{a3} - k_a}{\Delta k} \end{aligned} \right\} \quad (41)$$

Of course, the three points should be rather widely spaced, and errors can be reduced by choosing several sets of three points at random.

RESPONSE OF A HOT-WIRE IN AN ISOTHERMAL FLOWING MIXTURE
WITH VELOCITY AND CONCENTRATION FLUCTUATIONS

Voltage Fluctuation

Start again with King's equation

$$\frac{i^2 R}{R - R_a} = \frac{a l}{R_0 \alpha} k + \frac{b l}{R_0 \alpha} (d c_p \rho k U)^{1/2}$$

and introduce small fluctuations in velocity and concentration:

$$\left. \begin{aligned} R &= \bar{R} + r & \rho &= \bar{\rho} + \rho' \\ k &= \bar{k} + k' & U &= \bar{U} + u \\ c_p &= \bar{c}_p + c_p' & \Gamma &= \bar{\Gamma} + \gamma \end{aligned} \right\} \quad (42)$$

From equation (33),

$$\left. \begin{aligned} \rho' &= \gamma \Delta \rho \\ c_p' &= \gamma \Delta c_p \\ k' &= \frac{df(\bar{\Gamma})}{d\bar{\Gamma}} \gamma \Delta k \end{aligned} \right\} \quad (43)$$

Substituting equation (38) into King's equation, using appropriate binomial expansions, neglecting products of small quantities, and subtracting the averaged form of King's equation yields

$$-\frac{i^2 R_a r}{(\bar{R} - R_a)^2} = \bar{A} \frac{k'}{\bar{k}} + \frac{\bar{B}}{2} \bar{U}^{1/2} \left(\frac{c_p'}{\bar{c}_p} + \frac{\rho'}{\bar{\rho}} + \frac{k'}{\bar{k}} + \frac{u}{\bar{U}} \right) \quad (44)$$

But, $e = ir$; $\bar{B}$ can be replaced according to the averaged equation; and ρ' , c_p' , and k' are given in equation (43). Thus,

$$e = -\frac{(\bar{R} - R_a)^2}{1R_a} \left[\bar{A} \bar{\Gamma} \frac{df}{d\bar{\Gamma}} \frac{\Delta k}{\bar{k}} + \frac{1}{2} \left(\frac{1^2 \bar{R}}{\bar{R} - R_a} - \bar{A} \right) \left(\frac{\Delta c_p}{\bar{c}_p} + \frac{\Delta \rho}{\bar{\rho}} + \frac{df}{d\bar{\Gamma}} \frac{\Delta k}{\bar{k}} \right) \bar{\Gamma} \right] \frac{\gamma}{\bar{\Gamma}} \\ - \frac{(\bar{R} - R_a)^2}{21R_a} \left(\frac{1^2 \bar{R}}{\bar{R} - R_a} - \bar{A} \right) \frac{u}{\bar{U}} \quad (45)$$

A more convenient form is obtained by substituting

$$\frac{\Delta c_p}{\bar{c}_p} \bar{\Gamma} = \frac{\Delta c_p \bar{\Gamma}}{c_{pa} + \bar{\Gamma} \Delta c_p} = \frac{1}{1 + \frac{c_{pa}}{\bar{\Gamma} \Delta c_p}}$$

$$\frac{\Delta \rho}{\bar{\rho}} \bar{\Gamma} = \frac{1}{1 + \frac{\rho_a}{\bar{\Gamma} \Delta \rho}}$$

$$\frac{\Delta k}{\bar{k}} \bar{\Gamma} = \frac{1}{\frac{f}{\bar{\Gamma}} + \frac{k_a}{\bar{\Gamma} \Delta k}}$$

Also, the $\frac{\Delta k}{\bar{k}}$ -terms can be combined. Then the final form of the equation giving the instantaneous voltage fluctuation due to concentration and velocity fluctuations is

$$e = -\frac{(\bar{R} - R_a)^2}{21R_a} \left[\left(\frac{1^2 \bar{R}}{\bar{R} - R_a} + \bar{A} \right) \frac{df}{d\bar{\Gamma}} \frac{k_a}{\bar{\Gamma} + \frac{k_a}{\bar{\Gamma} \Delta k}} + \left(\frac{1^2 \bar{R}}{\bar{R} - R_a} - \bar{A} \right) \left(\frac{1}{1 + \frac{c_{pa}}{\bar{\Gamma} \Delta c_p}} + \frac{1}{1 + \frac{\rho_a}{\bar{\Gamma} \Delta \rho}} \right) \frac{\gamma}{\bar{\Gamma}} \right] \\ - \frac{(\bar{R} - R_a)^2}{21R_a} \left(\frac{1^2 \bar{R}}{\bar{R} - R_a} - \bar{A} \right) \frac{u}{\bar{U}} \quad (46)$$

For $\bar{\Gamma} = 0$ or $\frac{\gamma}{\bar{\Gamma}} = 0$, equation (46) reduces to equation (3), the response equation for the simple anemometer. In general, for concentration

and velocity fluctuations of the same order of magnitude, none of the terms in equation (46) is negligible. Typical values are given in appendix B.

As mentioned in the previous section, investigations on material diffusion in turbulent flow will first be concerned with the mixing of two gases of equal densities, in which case $\Delta\rho = 0$.

Time Constant

From the section CONVENTIONAL HOT-WIRE-ANEMOMETER RESPONSE EQUATIONS,

$$\frac{4.2ms}{R_0\alpha} \frac{dR}{dt} = i^2 R - (A + B\sqrt{U})(R - R_a) \quad (1a)$$

With equation (2), this again gives

$$\frac{4.2ms}{R_0\alpha} \frac{dR}{dt} = \frac{i^2 R_a (R_e - R)}{R_e - R_a} \quad (4)$$

which is equivalent to equation (4) of reference 2. Hence, with the usual restriction of small fluctuations, the same approximation is used in the denominator of the right side:

$$R_e - R_a = (\bar{R} - R_a) + (R_e - \bar{R}) \approx \bar{R} - R_a$$

so that equation (4) becomes approximately

$$\frac{4.2ms}{R_0\alpha} \frac{dR}{dt} = \frac{i^2 R_a (R_e - R)}{\bar{R} - R_a}$$

or

$$\frac{d}{dt}(R - \bar{R}) + \frac{1}{M''}(R - R_e) = 0$$

where

$$M'' = \frac{4.2ms(\bar{R} - R_a)}{i^2 R_a R_0\alpha}$$

just as for the simple anemometer. Of course, the numerical value of the time constant in this case is a function of both local mean velocity and mean concentration.

MEASUREMENT PROCEDURES FOR VELOCITY AND CONCENTRATION FLUCTUATIONS

Single Normal Wire

The voltage fluctuation for a simple hot-wire set normal to the mean flow direction (equation (46)) may be written as

$$e = \mu \frac{\gamma}{\bar{\Gamma}} + \delta \frac{u}{\bar{U}} \quad (46a)$$

The general procedure to be followed in using equation (46) to

determine $\overline{\left(\frac{\gamma}{\bar{\Gamma}}\right)^2}$, $\overline{\left(\frac{u}{\bar{U}}\right)^2}$, and $\overline{\left(\frac{\gamma u}{\bar{\Gamma} \bar{U}}\right)}$ is similar to that for the temperature-fluctuation problem, with an essential difference; namely, in this case the three readings of e^2 taken at a given point in the flow cannot ordinarily be made simply with the same hot-wire operating at different temperatures. This is due, of course, to the fact that μ and δ vary proportionally with changes in wire current (or temperature) at a given point in the flow. Instead, three wires with nonproportional mean calibration constants must be used. According to King's equation, this is possible only with three wires of differing diameter. However, in actual practice, the ordinary differences in A and B between two wires of the same nominal diameter may provide sufficient variation for this method.

It should also be pointed out that, since A is a function of the radiation loss, it may be possible to make at least two of the three necessary readings by using one hot-wire at widely differing temperatures.

Thus, the equations to be solved are completely analogous to equation (25) and need not be written out.

Possible Short Cuts

In general, it is not possible to make $\mu \gg \delta$ or $\mu \ll \delta$ for ordinary ranges of concentration and velocity. However, it may be possible to use two close parallel wires, the voltage of which is subtracted before

entering the amplifier. Then, if $\mu_1 = \mu_2$, the difference will measure $\left(\frac{u}{U}\right)^2$, while, if $\delta_1 = \delta_2$, the difference will measure $\left(\frac{\gamma}{F}\right)^2$.

As pointed out under MEASUREMENT PROCEDURES FOR VELOCITY AND TEMPERATURE FLUCTUATIONS, the wires must, of course, be far enough apart to have negligible sensitivity to v' .

The conditions necessary for making $\mu_1 = \mu_2$ without simultaneously making $\delta_1 = \delta_2$ are so complex as to seem impracticable. The inverse condition is a bit simpler:

$$\frac{(\bar{R}_1 - R_{a1})^2}{i_1 R_{a1}} B_1 = \frac{(\bar{R}_2 - R_{a2})^2}{i_2 R_{a2}} B_2 \quad (47)$$

But even this is extremely difficult to employ. In fact, perhaps the simplest conceivable assumptions which may still be satisfied by a pair of wires may be considered: Let $B_1 = B_2$ and $R_{a1} = R_{a2}$ with the specific condition that $A_1 \neq A_2$, since equal A 's together with the other conditions would also make $\mu_1 = \mu_2$. Now equation (47) becomes

$$\frac{i_1}{i_2} = \frac{(\bar{R}_1 - R_a)^2}{(\bar{R}_2 - R_a)^2} \quad (48)$$

In order to predict the necessary currents, $\bar{R}_1$ and $\bar{R}_2$ must be eliminated. This leads to

$$\frac{i_2^3}{i_1^3} = \frac{(\bar{A}_2 - i_2^2 + \bar{B} \sqrt{U})^2}{(\bar{A}_1 - i_1^2 + \bar{B} \sqrt{U})^2}$$

which should be solved explicitly for $i_2(i_1)$. This is not generally possible. Therefore, if this approach is to be used for direct measurement of $\left(\frac{\gamma}{F}\right)^2$, perhaps a feasible technique would be to satisfy equation (48)

by trial and error. Also, it should be noted that, since this assumes nearly identical wires operating at different temperatures, the time constants will be different, and separate matched amplifiers will be necessary.

Directionally Sensitive Meters

The general approach and procedure is completely analogous to that described for simultaneous temperature and velocity fluctuations.

Assuming a symmetrical meter,

$$\left. \begin{aligned} e_1 &= \mu \frac{\gamma}{\bar{F}} + \delta \frac{u}{\bar{U}} + \epsilon \frac{v}{\bar{U}} \\ e_2 &= \mu \frac{\gamma}{\bar{F}} + \delta \frac{u}{\bar{U}} - \epsilon \frac{v}{\bar{U}} \end{aligned} \right\} \quad (49)$$

These give

$$\overline{\left(\frac{v}{\bar{U}}\right)^2} = \frac{\overline{(e_1 - e_2)^2}}{4\epsilon^2} \quad (50)$$

and

$$\overline{e_1^2} - \overline{e_2^2} = 4\mu \epsilon \overline{\left(\frac{\gamma v}{\bar{F}\bar{U}}\right)} + 4\delta \epsilon \overline{\left(\frac{uv}{\bar{U}^2}\right)} \quad (51)$$

Thus the turbulent shear and lateral diffusion of material are obtained by repeating equation (51) with an appropriately different instrument and solving the resulting pair of simultaneous equations. Of course, the terms in equation (51) are proportional to the turbulent shear and material transfer only if the density fluctuations can be neglected (appendix D).

Correlations between Different Points

An approach identical with that for the temperature and velocity fluctuations gives, instead of equation (32),

$$\overline{(e_1 + e_2)^2} - \overline{(e_1 - e_2)^2} = 4\mu^2 \overline{\left(\frac{\gamma_1 \gamma_2}{\bar{F}^2}\right)} + 4\mu \delta \left(\overline{\frac{\gamma_1 u_2}{\bar{F}\bar{U}}} + \overline{\frac{\gamma_2 u_1}{\bar{F}\bar{U}}} \right) + 4\delta^2 \overline{\left(\frac{u_1 u_2}{\bar{U}^2}\right)} \quad (52)$$

and the three quantities in parentheses on the right side can be obtained by repeating the readings with two other appropriately different pairs of hot wires.

Again, unlike the case in the section MEASUREMENT PROCEDURES FOR VELOCITY AND TEMPERATURE FLUCTUATIONS, neither $\frac{\overline{\gamma_1 \gamma_2}}{\bar{\Gamma}^2}$ nor $\frac{\overline{u_1 u_2}}{\bar{U}^2}$ can be measured directly.

The correlation between v fluctuations at two different points is determinable as outlined in the aforementioned section.

Spectrum

Since, in general, the value of μ cannot be made either very large or very small compared to the value of δ , no direct measurement of either the γ' spectrum or the u' spectrum can be made.

In the case of gases whose densities and viscosities are not too widely different, the u' spectrum in the mixing gases is probably quite close to that in a similar flow of either component alone, for which case it is easily measurable. A comparison of this u' spectrum in the pure gas with an e' spectrum measured in the mixture may, at least, give a rough idea of the direction in which the γ' spectrum varies as compared with u' .

Again it should be noted that the v' spectrum is measurable directly in a flowing gas mixture with concentration fluctuations.

CONCLUDING DISCUSSION

In general, the errors inherent in these new applications of the hot-wire anemometer are the same as for the simple anemometer (references 2, 26, 27, and others) and hence need not be discussed here.

Perhaps the only additional source of error arises simply from the added complication of many of these measurements. This influences the result in two ways: (a) The final numerical result is obtained from more separate readings of fluctuating quantities than is the case in ordinary anemometric applications and (b) the appreciably greater time required for measurement at each point in the flow results in greater difficulty in maintaining the steady-state condition of the apparatus, both aerodynamic and electrical.

It should also be noted that the hot-wire length corrections will be different for the different quantities, since each case must be computed from the correlation function of the physical quantity in question.

The details of the present analysis assume the validity of King's equation in its conventional form. The accuracy with which this equation predicts the influence of velocity changes has been verified countless times; preliminary attempts to verify or disprove the rest of this equation (appendix A) have been indecisive. Considerably more careful measurements must be made, both in different pure gases and in a pure gas at various temperatures. However, the general method presented here is equally applicable to a possibly more accurate heat-transfer equation that may be found in the future.

The possible usefulness of a method for such detailed investigation of nonkinematic quantities in a turbulent flow is clear. For basic research, most of the measurable statistical quantities should be of interest; for some practical problems, perhaps the direct measurement of mean distributions and of turbulent transfer will be paramount.

In principle, there appears to be no necessity for restriction of the method to measurement in a flow with only two simultaneous fluctuating quantities. However, the complication that arises upon consideration of simultaneous fluctuations in velocity, temperature, and concentration would appear to render this generalization impracticable. The idea of being able to make such measurements in a turbulent combustion zone is naturally of interest. However, this extremely important problem presents at least two certain major difficulties: (a) In reasonable density ranges, the flame temperatures are excessive and (b) the intermediate stages of most chemical reactions would involve dealing with a mixture of several components instead of just two. In addition to these difficulties, the possible catalytic action of the instrument in the reaction zone might also be noted as well as the probability that the hot-wire might act as an igniter.

In casting about for possible methods of making direct measurement of the various kinematic quantities in a flowing gas mixture, the possibility should not be ignored of finding a pair of gases whose densities, specific heats, and thermal conductivities are so related that the net response to a concentration fluctuation is negligible. It can be seen from the numerical values in appendix B that such a situation is not beyond the realm of possibility.

California Institute of Technology
Pasadena, Calif., June 10, 1947

APPENDIX A

REMARKS ON KING'S EQUATION

General

As pointed out by McAdams (reference 28), the forced-convection term in King's equation can be written in terms of the product of a Reynolds number and a Prandtl number, so that the rate of heat loss from the wire is

$$i^2 R = k(T - T_a)(a' + b' \sqrt{R\sigma}) \quad (A1)$$

where R under the radical is Reynolds number.

Inserting an extra d and introducing a heat-transfer coefficient h and the Nusselt number $\eta = \frac{hd}{k}$, McAdams writes this in the form

$$\eta = \kappa_1 + \kappa_2 \sqrt{R\sigma} \quad (A2)$$

where R is Reynolds number.

Conventionally, the first term in King's equation is described as including radiation and free convection, while the second term is due to forced convection. The latter appears to be quite reasonable, but it is apparent that free-convection effects, if appreciable, cannot enter the heat-transfer equation simply additively; the free-convection phenomenon is complex, and in most current applications the direction of buoyancy-induced velocity is perpendicular to the main flow. The form of A , as given by King, gives no clue as to its physical origin, although it is physically obvious that radiation must be included. The question is perhaps further clouded by the fact that measurements of the zero-velocity point in constant-resistance hot-wire calibrations seem to agree rather well with the A obtained by the extrapolation of the points measured at relatively large flow velocities.

It would seem that, in view of the extended possibilities of the hot-wire anemometer as described in the present report, a complete reexamination of the fundamental heat-transfer equation for the flow past a heated cylinder at small Reynolds numbers is in order, with particular emphasis on explicit inclusion of all the significant physical quantities. For example, if A actually does represent a free-convection term, it should include a Grashof number, which generally has a rather complicated variation with temperature.

For the purposes of the present work, preliminary attempts have been made to check the validity of King's equation. These are described briefly in the following paragraphs.

Mean Calibration of Hot Wire in Air at Various Temperatures

Because of the difficulty of obtaining a series of velocities at each of several temperatures with the present equipment, single measurements were made at various velocities and temperatures, keeping $\bar{R} - \bar{R}_a = \text{Constant}$. Since King's $\bar{B}$ is relatively insensitive to temperature changes and since this forced-convection term seems physically sound, the measured points were used to compute the intercept $\bar{A}$ on the assumption that the theoretical variation of $\bar{B}(\bar{T}_a)$ is correct. Thus, an $\bar{A}$ was computed from each measured point instead of from a complete faired calibration curve at each temperature. In this computation, the data of Taylor and Johnston (reference 29) for the thermal conductivity of dry air were used. The air was, in fact, fairly humid, but here only the relative values are important, and an error in absolute magnitude of k would have no effect. The specific heat was assumed constant ($c_p = 0.24$) over the temperature range, in accordance with the International Critical Tables (reference 23). For the density variation, the air was assumed to be a perfect gas.

The results (fig. 3) show considerable scatter because this form of presentation involves small differences between two relatively large quantities, one of which has appreciable scatter itself. From the figure it appears that $\bar{A}(\bar{T}_a)$ probably varies more slowly than $k(\bar{T}_a)$; however, the large amount of scatter precludes the deduction of an alternative, empirical expression. The agreement appears to be sufficiently good to justify the use of King's equation in the present, fairly rough measurements of velocity and temperature fluctuations in a free jet.

Comparison of Mean Calibrations in Air and Carbon Dioxide

In a further attempt to check King's equation, a hot-wire was calibrated in both air and carbon dioxide. Then, with the assumption that the forms of King's A and B are correct, the carbon dioxide calibration was computed from the air calibration. The curves are given in figure 4.

For the computation of A_{CO_2} and B_{CO_2} from A_{air} and B_{air} , the thermal conductivities, which correspond to those of pure gases, were taken from references 29 and 30. The specific heats and densities were taken from reference 23, and both gases were assumed to be perfect gases.

From figure 4 it can be seen that, although the agreement between measured and computed B_{CO_2} is good, the two values of A_{CO_2} are relatively far apart.

The principal possible source of error in the determination of A_{CO_2} and B_{CO_2} from the air calibration is the use of thermal conductivities

measured in relatively pure gases. The laboratory air undoubtedly contained appreciable amounts of carbon dioxide and water vapor (the measurements were made on a cloudy day); the jet carbon dioxide was a commercial type, stated as $99\frac{1}{2}$ percent pure, probably containing air, water, and other impurities. Small amounts of impurities may have an appreciable effect upon thermal conductivity, much larger, for example, than the effect upon density and specific heat. Unfortunately, insufficient experimental data exist on the thermal conductivity of gas mixtures, and the theoretical approach has proved to be tremendously complicated, even the first approximation for monatomic gases (reference 24). Thus, it would be extremely difficult to estimate the error due to impurities, even if a quantitative analysis were available. It is concluded that the measurements should be repeated with pure gases or that the actual thermal conductivities should be measured.

APPENDIX B

TYPICAL NUMERICAL VALUES OF TERMS IN RESPONSE EQUATIONS

Temperature and Velocity Fluctuations

The complete equation (18) can be written in the form

$$\frac{i\bar{R}_a}{(\bar{R} - \bar{R}_a)^2} \vartheta = \underbrace{\left(\bar{A} + \bar{B} \sqrt{\bar{U}} \right) \frac{R_0 \alpha}{(\bar{R} - R_a)} \vartheta}_{(a)} - \underbrace{n \frac{\bar{A}}{\bar{k}} \vartheta}_{(b)}$$

$$- \underbrace{\frac{\bar{B}}{2} \sqrt{\bar{U}} \left(\frac{n}{\bar{k}} - \frac{1}{\bar{T}_a} \right) \vartheta}_{(c)} - \underbrace{\frac{\bar{B}}{2} \frac{u}{\sqrt{\bar{U}}}}_{(d)} \quad (B1)$$

It is of interest to compare the relative magnitudes of the four terms as indicated.

Assume the following typical numerical values:

$$\begin{aligned} \bar{U} &= 1000 \text{ cm/sec} & d &= 0.000611 \text{ cm} \\ \bar{\theta} &= 100^\circ \text{ C} & R_0 &= 11.0 \Omega \\ T_r &= 290^\circ \text{ abs.} & l &= 0.30 \text{ cm} \\ \frac{u}{\bar{U}} = \frac{\vartheta}{\bar{\theta}} &= 0.20 & i &= 0.040 \text{ amp} \\ & & \alpha &= 0.0037 \text{ (for platinum wire)} \end{aligned}$$

$$\bar{A} = 0.0035$$

$$\bar{B} = 0.00010$$

$$\bar{k} = 7.55 \times 10^{-5} \text{ (reference 29)}$$

$$\bar{p} = 7.00 \times 10^{-4}$$

$$\bar{c}_p = 0.25$$

$$n = 1.62 \times 10^{-7} \text{ (reference 29)}$$

These values give

$$R_r = R_0 [1 + \alpha(T_r - 273)] = 11.69\Omega$$

$$\bar{R}_a = R_0 [1 + \alpha(\bar{T}_a - 273)] = 15.76\Omega$$

and, from the equation for thermal equilibrium,

$$\bar{R} = \bar{R}_a + \frac{i^2 \bar{R}_a}{\bar{A} - i^2 + \bar{B} \sqrt{\bar{U}}} = 20.74\Omega$$

The following numerical values are now obtained for the terms in equation (B1):

$$(a) = 10.90 \times 10^{-4}$$

$$(b) = 1.22 \times 10^{-4}$$

$$(c) = -0.13 \times 10^{-4}$$

$$(d) = 3.16 \times 10^{-4}$$

Term (c) is only a little more than 1 percent of term (a) and is apparently negligible for all anticipated operating conditions.

Concentration and Velocity Fluctuations in

Mixtures of Air and Carbon Dioxide

Since there is no information available on precise measurements of the thermal conductivity of mixtures of air and carbon dioxide, it is assumed for the purposes of this calculation that the variation is linear; that is, $f(\bar{T}) = \bar{T}$. Then equation (46) can be written in the form

$$\begin{aligned}
 - \frac{2iR_a}{(\bar{R} - R_a)^2} e = & \underbrace{\frac{(2\bar{A} + \bar{B}\sqrt{\bar{U}})}{k_a} \left(\frac{\gamma}{\bar{T}} \right)}_{(a)} + \underbrace{\frac{\bar{B}\sqrt{\bar{U}}}{c_{p_a}} \left(\frac{\gamma}{\bar{T}} \right)}_{(b)} \\
 & + \underbrace{\frac{\bar{B}\sqrt{\bar{U}}}{1 + \frac{\rho_a}{\bar{T} \Delta p}} \left(\frac{\gamma}{\bar{T}} \right)}_{(c)} + \underbrace{\bar{B} \frac{u}{\sqrt{\bar{U}}}}_{(d)}
 \end{aligned} \tag{B2}$$

Assume the following typical numerical values:

$\bar{U} = 1000 \text{ cm/sec}$	$l = 0.30 \text{ cm}$	$\rho_a = 1.165 \times 10^{-3}$
$\bar{\Gamma} = 0.50$	$i = 0.040 \text{ amp}$	$\rho_g = 1.785 \times 10^{-3}$
$T_r = T_a = 290^\circ \text{ abs.}$	$\alpha = 0.0037$	$c_{pa} = 0.25$
$\frac{u}{\bar{U}} = \frac{\gamma}{\bar{\Gamma}} = 0.20$	$\bar{A} = 0.0020$	$c_{pg} = 0.199$
$d = 0.000611 \text{ cm}$	$\bar{B} = 0.00010$	$\Delta k = -2.13 \times 10^{-5}$
$R_0 = 11.0\Omega$	$k_a = 5.93 \times 10^{-5}$	$\Delta p = 0.620 \times 10^{-3}$
	$k_g = 3.80 \times 10^{-5}$	$\Delta c_p = -0.051$

These give

$$(a) = -3.14 \times 10^{-4}$$

$$(b) = -0.72 \times 10^{-4}$$

$$(c) = 1.33 \times 10^{-4}$$

$$(d) = 3.16 \times 10^{-4}$$

and none of the terms is negligible.

APPENDIX C

COMPENSATION RESISTANCE FOR SIMPLE RESISTANCE THERMOMETER

With appreciable heating current, the ordinary anemometric expression for the time constant can be used:

$$M = \frac{4.2ms}{R_0\alpha} \frac{(\bar{R} - \bar{R}_a)}{i^2 \bar{R}_a} \quad (C1)$$

which is more convenient than the equivalent form

$$M' = \frac{4.2ms}{R_0\alpha} \frac{1}{(\bar{A} - i^2 + \bar{B} \sqrt{\bar{U}})} \quad (C2)$$

The compensation resistance is then given by

$$R_c = \frac{L}{M} \quad (C3)$$

where L is the inductance of the coil in the compensation network.

However, in order to measure temperature fluctuation level directly, it is convenient to operate with negligible heating current. Then $(\bar{R} - \bar{R}_a)$ becomes too difficult to measure in a turbulent flow, and the limit must be reached with equation (C2) rather than with equation (C1). Then

$$M' = \frac{4.2ms}{R_0\alpha} \frac{1}{(\bar{A} + \bar{B} \sqrt{\bar{U}})} \quad (C4)$$

where, as in equation (C2), the values of $\bar{A}$ and $\bar{B}$ to be used are the average values corresponding to temperature $\bar{T}_a$.

In the actual process of measurement, perhaps the shortest method is to determine $(\bar{A} + \bar{B} \sqrt{\bar{U}})$ by a measurement of $\frac{i^2 \bar{R}}{\bar{R} - \bar{R}_a}$ with the wire appreciably heated, since this reading is also necessary for the determination of mean velocity.

APPENDIX D

TRANSFER IN TURBULENT SHEAR FLOW WITH DENSITY FLUCTUATIONS

In the sections on measurement procedures, methods are given for the determination of shear, heat transfer, and material transfer by measurement with a directionally sensitive hot-wire meter. The procedures given actually lead to the quantities $\overline{uv}$, $\overline{\theta v}$, and $\overline{\gamma v}$. These, of course, are proportional to the respective transfer rates only in cases for which the density and specific heat fluctuations are sufficiently small, in which cases the local mean density can be used to give

$$\left. \begin{array}{ll} \text{Shear:} & \tau = -\overline{\rho uv} \\ \text{Heat transfer:} & Q = -\overline{\rho c_p \theta v} \\ \text{Material transfer:} & D = -\overline{\rho \gamma v} \end{array} \right\} \quad (D1)$$

Since the density fluctuations have been taken into account for the computation of the hot-wire response, it may be well to investigate the limitations introduced by the use of equation (D1).

Temperature and Velocity Fluctuations

The general expression for turbulent shear is

$$\tau = -\overline{\rho UV} \quad (D2)$$

but

$$\left. \begin{array}{l} \rho = \overline{\rho} + \rho' \\ U = \overline{U} + u \\ V = v \\ \theta = \overline{\theta} + \theta' \end{array} \right\} \quad (D3)$$

For turbulent shear flow with temperature gradient, it may reasonably be assumed that $\frac{u}{\overline{U}}$, $\frac{v}{\overline{U}}$, and $\frac{\theta'}{\overline{\theta}}$ are the same order of magnitude; in fact, in the previous discussion they are assumed small. It is of interest to find the restriction on $\overline{\theta}$ such that equation (D1) may be applied with at least the same accuracy as may be expected from the measuring technique.

With equation (D3), equation (D2) becomes

$$\tau = -\bar{\rho}\bar{u}\bar{v} - \bar{U}\bar{\rho}'\bar{v} - \bar{\rho}'\bar{u}\bar{v}$$

and the third term is clearly negligible.

Since the density fluctuation is due to temperature fluctuation, from equation (17),

$$\rho' = -\bar{\rho} \frac{\partial}{\partial T_a}$$

Therefore,

$$\tau = -\bar{\rho}\bar{u}\bar{v} + \frac{\bar{\rho}\bar{U}}{\bar{T}_a} \bar{\partial v} \quad (D4)$$

Hence, for equation (D1) to be valid,

$$\bar{u}\bar{v} \gg \frac{\bar{U}}{\bar{T}_a} \bar{\partial v}$$

or,

$$\frac{\bar{u}\bar{v}}{\bar{U}^2} \gg \frac{\bar{\partial v}}{\bar{\theta U}} \frac{\bar{\theta}}{\bar{T}_a} \quad (D5)$$

If $\frac{\bar{u}\bar{v}}{\bar{U}^2}$ and $\frac{\bar{\partial v}}{\bar{\theta U}}$ are the same order of magnitude, this requires that

$$\frac{\bar{\theta}}{\bar{T}_a} \ll 1 \quad (D6)$$

If this condition is not satisfied with sufficient accuracy, the turbulent shear is calculable from equation (D4) after $\bar{u}\bar{v}$ and $\bar{\partial v}$ have been obtained in accordance with equation (30).

The turbulent heat transfer is

$$Q = -c_p \bar{\rho} \bar{\theta} \bar{v} \quad (D7)$$

but, with equation (D3) and with $c_p = \text{Constant}$,

$$Q = -c_p \bar{\rho} \bar{\partial v} - c_p \bar{\theta} \bar{\rho}' \bar{v} - c_p \bar{\rho}' \bar{\partial v}$$

where the third term can be neglected. Substituting for ρ' yields

$$Q = -c_p \bar{\rho} \bar{\theta} \bar{\gamma v} + c_p \bar{\rho} \frac{\bar{\theta}}{\bar{T}_a} \bar{\gamma v} \quad (D8)$$

so that, for equation (D1) to be valid, it is required that

$$\frac{\bar{\theta}}{\bar{T}_a} \ll 1 \quad (D6)$$

as before. If equation (D6) is not satisfied, the turbulent heat transfer is easily calculable from

$$Q = -c_p \bar{\rho} \left(1 - \frac{\bar{\theta}}{\bar{T}_a} \right) \bar{\gamma v} \quad (D9)$$

after $\bar{\gamma v}$ has been obtained from equation (30).

Concentration and Velocity Fluctuations

With equation (D2), equation (D3), and the expression for the density fluctuation,

$$\rho' = \gamma \Delta \rho$$

where $\Gamma = \bar{\Gamma} + \gamma$, there is obtained for the turbulent shear stress, approximately

$$\tau = -\bar{\rho} \bar{u v} - \bar{U} \Delta \rho \bar{\gamma v} \quad (D10)$$

If equation (D1) is to be reasonably accurate and if $\frac{\bar{u v}}{\bar{U}^2}$ is roughly equal to $\frac{\bar{\gamma v}}{\bar{\Gamma} \bar{U}}$, the following inequality must be satisfied:

$$\frac{\Delta \rho}{\bar{\rho}} \ll \frac{1}{\bar{\Gamma}} \quad (D11)$$

If equation (D11) is not satisfied, $\bar{u v}$ and $\bar{\gamma v}$ can be obtained from equation (51), and equation (D10) gives the shearing stress.

The turbulent material transfer is

$$D = -\bar{\rho} \bar{\Gamma v} \quad (D12)$$

Introducing the mean values and fluctuations and neglecting the triple product of the small fluctuating quantities as before, equation (D12) becomes

$$D = -\overline{\rho\gamma v} - \overline{\bar{\Gamma}\rho'v} \quad (D13)$$

But $\rho' = \gamma \Delta\rho$, so that

$$D = -(\bar{\rho} + \bar{\Gamma} \Delta\rho)\overline{\gamma v} \quad (D14)$$

and the condition for D in equation (D1) to be valid is again

$$\frac{\Delta\rho}{\bar{\rho}} \ll \frac{1}{\bar{\Gamma}} \quad (D11)$$

As before, equation (D14) may be used after $\overline{\gamma v}$ has been obtained from equation (51).

APPENDIX E

MIXTURES OF PERFECT GASES

In view of the sections discussing measurements in gas mixtures, it seems appropriate to include a brief review of pertinent relations occurring in the theory of mixtures of perfect gases. Extensive treatment is to be found, for example, in reference 31.

The following notation is used:

V	total volume (for most problems in fluid mechanics, unit volume is considered)
M_i	molecular weight of i th component
N_i	number of moles of i th component
p_i	partial pressure of i th component
ρ_i	density of i th component at pressure p_i
ρ_i^*	density of i th component at pressure p
R	universal gas constant
T	absolute temperature
p, ρ, M	characteristics of mixture (M , apparent molecular weight)

There follows immediately,

$$p = \sum_i p_i \quad (E1)$$

$$\rho_i = \frac{N_i M_i}{V} \quad (E2)$$

$$\rho_i^* = \rho_i \frac{p}{p_i} \quad (E3)$$

Also, the equation of state is satisfied for each component and for the mixture:

$$p_1 V = N_1 RT \quad (E4)$$

$$pV = \sum_i N_i RT \quad (E5)$$

or

$$p_1 = \frac{\rho_1}{M_1} RT \quad (E6)$$

and

$$p = \sum_i \frac{\rho_i}{M_i} RT = \frac{\rho}{M} RT \quad (E7)$$

$$\text{Since } \rho = \sum_i \rho_i,$$

$$\sum_i \frac{\rho_i}{M_i} = \frac{\sum_i \rho_i}{M}$$

so that

$$M = \frac{\sum_i \rho_i}{\sum_i \frac{\rho_i}{M_i}} \quad (E8)$$

Relative concentrations are of particular interest. Define the molal concentration of the i th component by

$$\gamma_i = \frac{N_i}{\sum_k N_k} \quad (E9)$$

But, from equations (E4) and (E5),

$$\frac{p_1}{p} = \frac{N_1}{\sum_{\kappa} N_{\kappa}}$$

so that

$$\gamma_1 = \frac{p_1}{p} \quad (\text{E10})$$

The following relation is of interest:

$$\rho(\rho_1^*, \gamma_1)$$

From equation (E3),

$$\rho = \sum_1 \rho_1 = \sum_1 \left(\rho_1^* \frac{p_1}{p} \right)$$

or

$$\rho = \sum_1 (\rho_1^* \gamma_1) \quad (\text{E11})$$

For a mixture of two gases, $\gamma_2 = 1 - \gamma_1$, and

$$\rho = \rho_2^* + (\rho_1^* - \rho_2^*) \gamma_1 \quad (\text{E12})$$

A mass concentration can be defined as

$$\lambda_1 = \frac{\rho_1}{\sum_{\kappa} \rho_{\kappa}} = \frac{\rho_1}{\rho} \quad (\text{E13})$$

A general expression for $\rho(\rho_1^*, \lambda_1)$ is rather complex. But equation (E13) can be transformed to

$$\lambda_1 = \frac{\rho_1^*}{\sum_{\kappa} \rho_{\kappa}} \gamma_1 \quad (\text{E14})$$

Specializing at once to a two-component mixture,

$$\rho = \frac{\rho_1^*}{\lambda_1} \gamma_1 \quad (\text{E15})$$

but, from equation (E12),

$$\gamma_1 = \frac{\rho - \rho_2^*}{\rho_1^* - \rho_2^*}$$

Substituting this into equation (E15) and solving for ρ yields

$$\rho = \frac{\rho_1^* \rho_2^*}{\rho_1^* + (\rho_2^* - \rho_1^*) \lambda_1} \quad (\text{E16})$$

which may be compared with equation (E12).

A relation between these two definitions of concentration is obtained by equating equations (E12) and (E16):

$$\lambda_1 = \frac{\rho_1^* \gamma_1}{\rho_2^* + (\rho_1^* - \rho_2^*) \gamma_1} \quad (\text{E17})$$

or

$$\gamma_1 = \frac{\rho_2^* \lambda_1}{\rho_1^* + (\rho_2^* - \rho_1^*) \lambda_1} \quad (\text{E18})$$

The relative significance of these two concentrations in the field of fluid mechanics merits consideration. In investigations of mass transfer in gas flow, the mass concentration would appear to be the more significant parameter. For example, in a comparison of lateral rates of diffusion of momentum, heat, and material in a particular turbulent shear flow, the λ distribution is to be compared with the velocity and temperature distributions.

On the other hand, a treatment of material diffusion prior to chemical reaction would obviously be more concerned with the molal concentration.

For hot-wire measurements, the simplicity of equation (E12) relative to equation (E16) recommends the determination of molal concentration and fluctuations first, and, if necessary, the use of equation (E17) to obtain mass concentration.

Of course, if $\rho_1^* = \rho_2^*$, a case of particular interest in the investigation of material diffusion in turbulent flow, the distinction vanishes.

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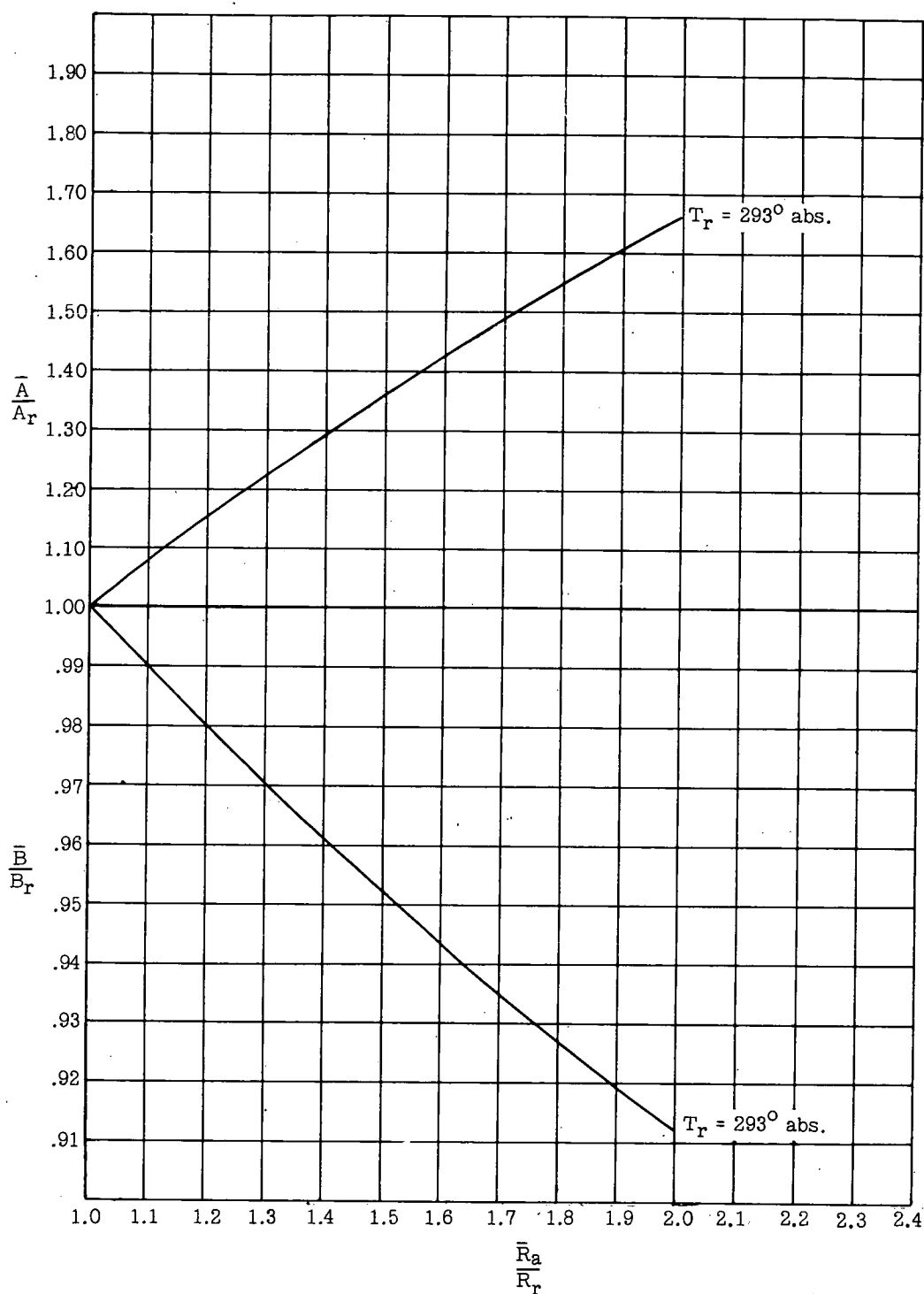


Figure 1.- Variation of hot-wire constants with air temperature. Physical constants from reference 23. R_r , resistance at reference temperature; $\bar{R}_a$, resistance at local temperature.

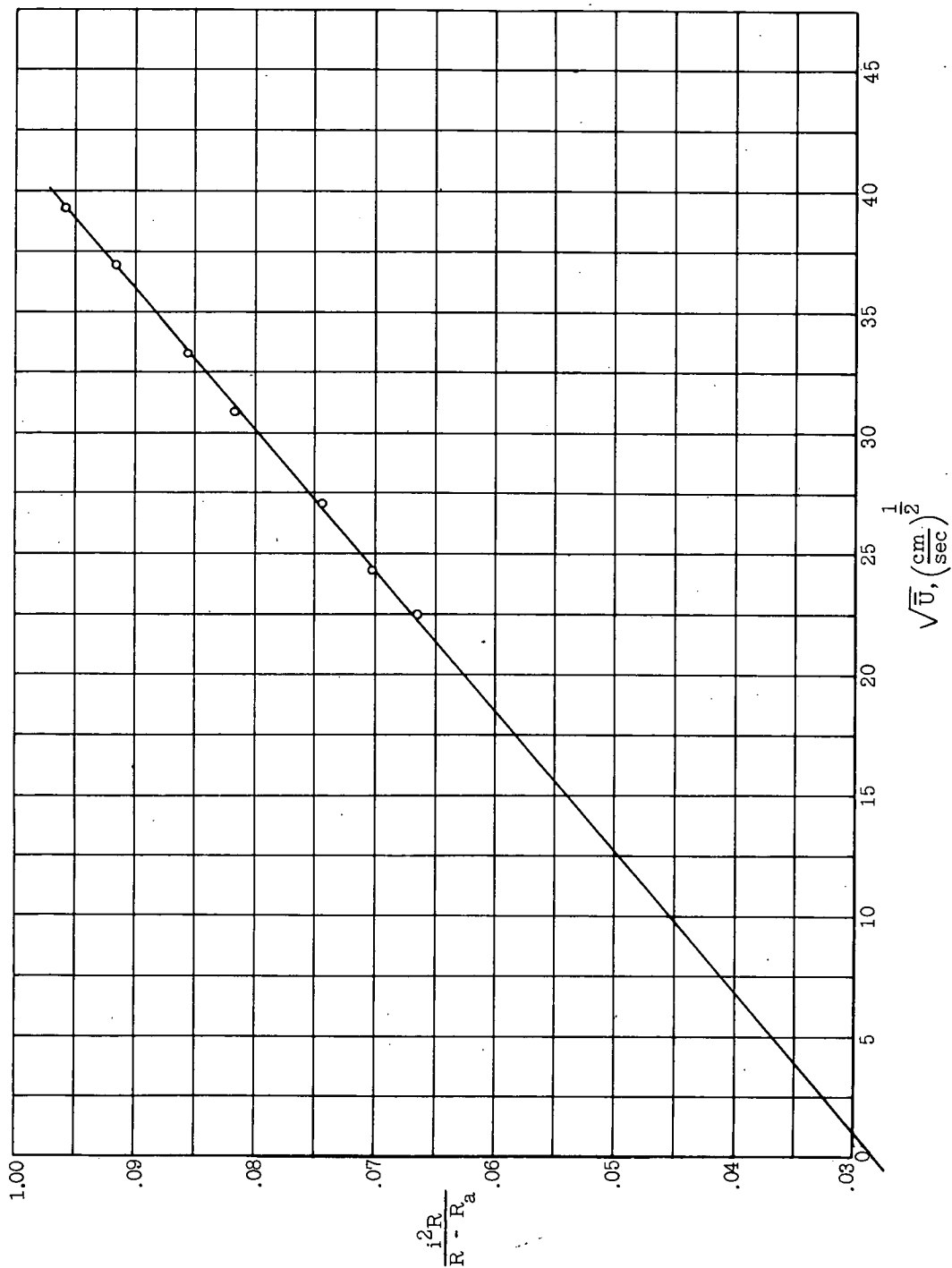


Figure 2.- Calibration of $\frac{1}{2}$ -mil platinum wire in air. $A = 0.00281$; $B = 0.00173$.

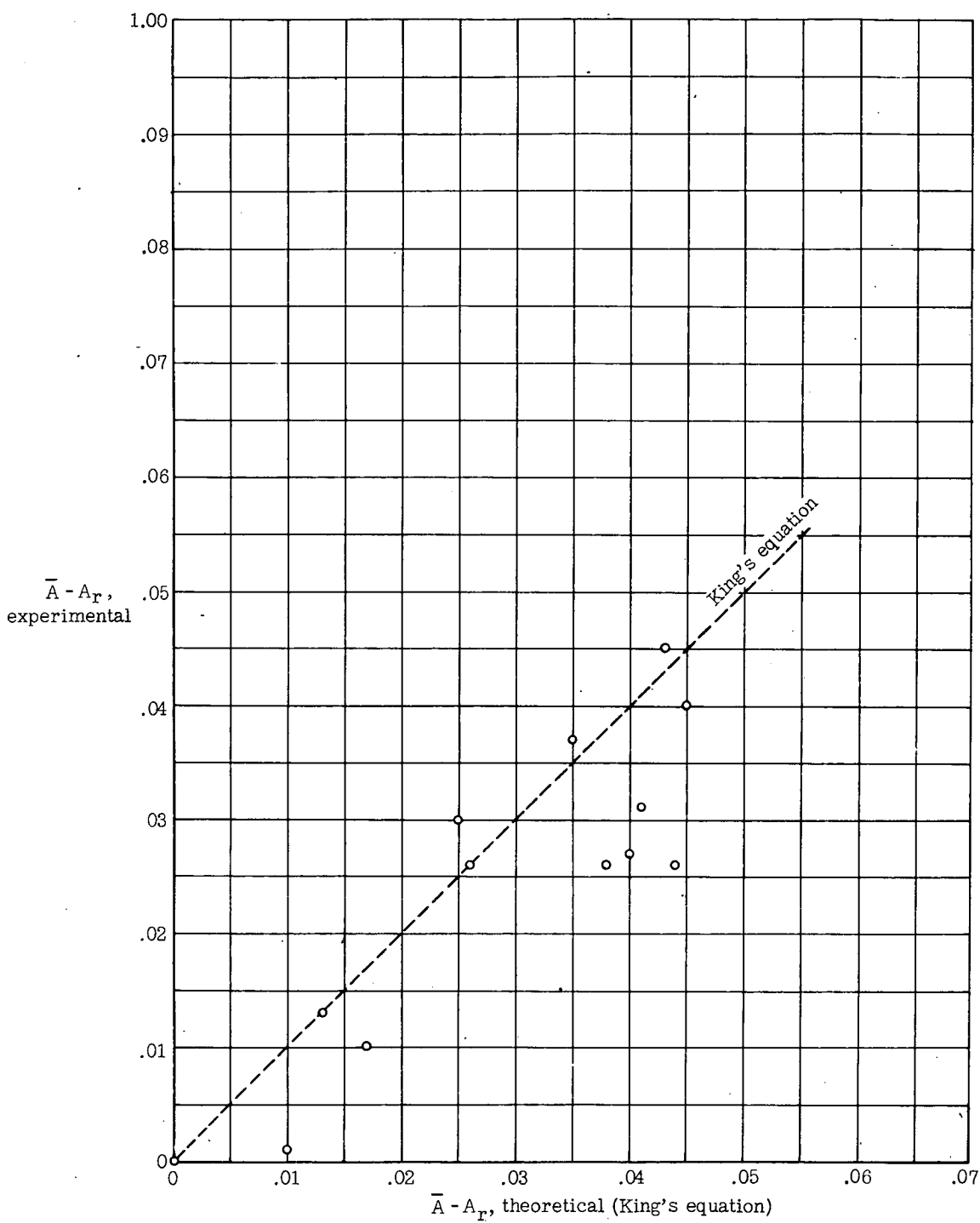


Figure 3.- Hot-wire calibration in heated air. Physical constants from references 23, 25, and 29: $20^{\circ} \text{ C} \leq T_a \leq 87^{\circ} \text{ C}$.

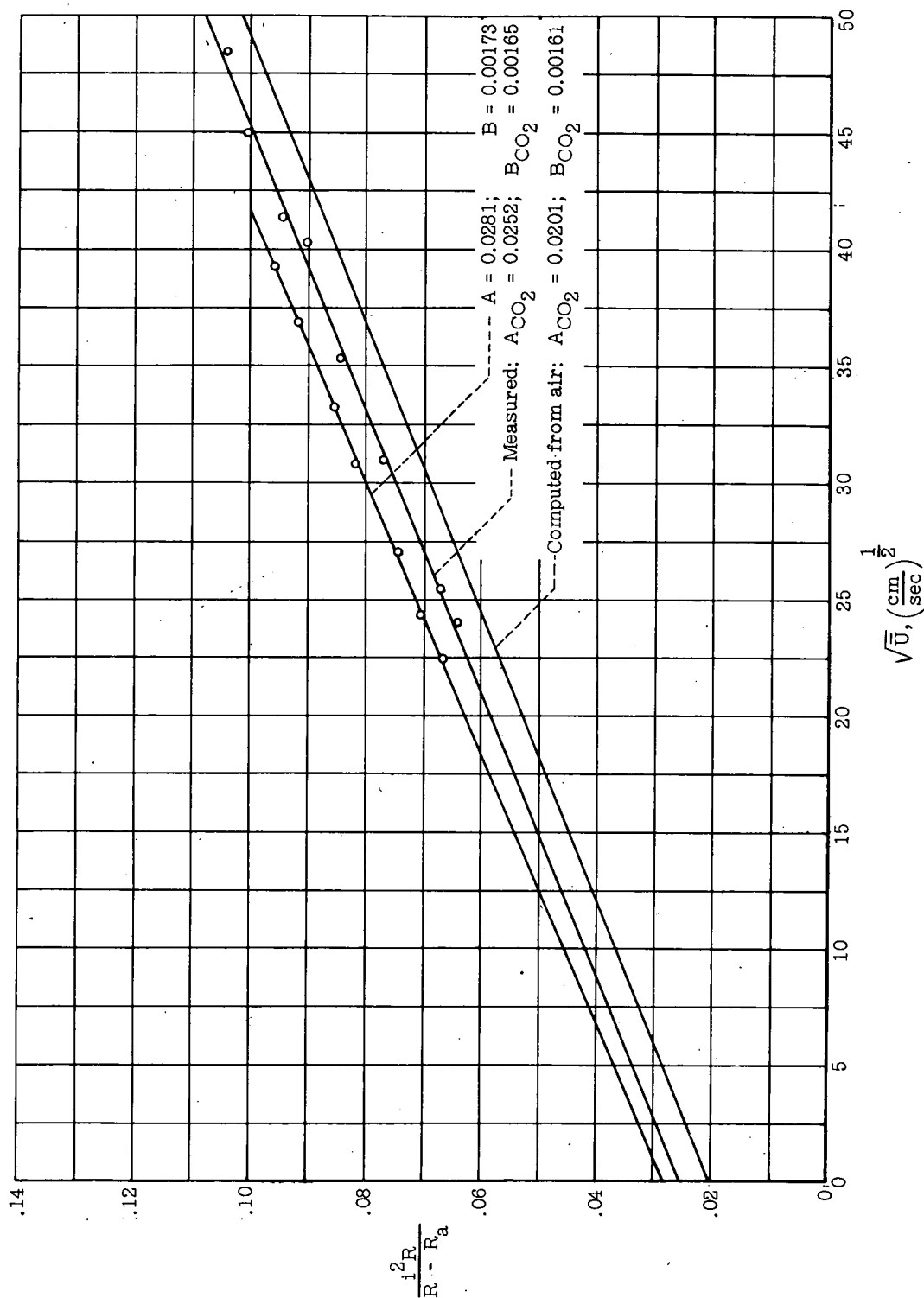


Figure 4.- Calibration of $\frac{1}{2}$ - mil platinum wire in carbon dioxide and air. Physical constants from references 23, 29, and 30.